PHILOSOPHICAL PRINCIPLES OF RELIGION.

PART II.

CONTAINING

The NATURE and KINDS of Infinites;
their Arithmetick and Uſes, and the Philoſophical
Principles of Revealed RELIGION,
now firſt publiſhed.

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THE PREFACE

To the Second PART.

BEfore the Reader takes the Trouble to enter on the Second Part, it is fit he be appriſed of a few Things, that may prevent his miſtaking my Meaning, in the more difficult Propoſitions, or may enable him to run through the whole with more Pleaſure.

I. The firſt Chapter is intended only, as an Introduction to the two ſubſequent, to the third eſpecially; and for clearing up and demonſtrating, in the moſt familiar and eaſy Way, the Nature and Properties of relative Infinites; the Foundations of the Arithmetick of Infinites, and the Structure built thereon, publiſhed by me in the former Edition of this Work, having been doubted of, or miſtaken by ſome. The whole now, I hope, is clear and unqueſtionable. I have borrowed a few Things from the third Chapter, for the Eaſe of thoſe, who either might not have Humour and Leiſure, or might not have applied themſelves ſufficiently to ſuch Studies, as to be able to go quite through that Chapter, and yet might be content to ſee this curious Speculation eſtabliſhed here.

II. The ſecond Chapter contains many Particulars of Weight and Moment. The Foundation is this: The ſupreme Creator of all Things, and the whole Syſtem of Creatures, from the higheſt Seraphim down to brute Matter, are here together conſidered as it were an infinite Cone, (like the Shadow of the dark ſide of the Earth, circumſcribed by the Light of the Sun in the empty Spaces of our Syſtem,) whoſe Baſe is the ſupreme and abſolute Infinite, the Origin of the Being and Faculties of all created Things; and its Body, is the whole Syſtem of Creatures, from the higheſt ſpiritual Intelligence, deſcending, in a perpetual Subordination, and continual Scale, down to brute Matter; or if there be any Creature lower than this. It is true, in this Metaphor or Reſemblance, the Baſe is to be ſuppoſed at an abſolutely infinite Diſtance, from the Body of the Cone, (as the Sun, whoſe Rays define the dark Cone of the Earth's Shadow, is diſtant from the Earth:) But then, as all the Sections in a Cone, parallel to the Baſe, are ſimilar to the Baſe, and to each other; So in this perpetual Scale of Creatures,

tures, conſidered in one View, together with their Creator, every Species and Set of Creatures is ſimilar to the Baſe, and to every other Species and Set of Creatures, from the higheſt to the loweſt, *i. e.* every Species and Set of Creatures of a higher Order, has the great Lineaments, and prominent Out-lines of their Baſe, the Origin of all Being and Perfections, more ſtrongly, clearly, and largely repreſented and expreſſed on, and by them; and every Species and Set of Creatures of a lower Order, has the ſame Lineaments and Characters repreſented and expreſſed on, and by them, but in a more weak, more faint, and more contracted Manner. And as Life, Activity, and Fecundity are among the moſt univerſal, primitive, and original Qualities of the Baſe, the Source, and Origin of all Being and Perfections: So every Species of Creatures, and each Individual of every Species, muſt in a higher or lower Degree, according to their Rank in the Scale of Exiſtence, partake of thoſe primitive and original Qualities. If this Principle and Foundation, thus ſhadowed out, may be allowed me, and ſure methinks, it is evident from the Nature of Things *à Priori*, from all Experiments and Obſervations hitherto made on our material Syſtem of Things *à Poſteriori*, and even from the moſt genuine and ſimple Reflections of our Minds within our ſelves: Then it will follow,

I. That there is a perpetual Analogy, (Phyſical not Mathematical) running on in a Chain, through the whole Syſtem of Creatures, up to their Creator.

II. That the Viſible are Images of the Inviſible, the Senſible of the Inſenſible, the Ectypal of the Archetyppal, the Creatures of the Creator, at an abſolutely infinite Diſtance.

III. That the Arguments from the Attributes of the Creator, to the Qualities of the Creature, with due regard to the abſolutely infinite Diſtance, is juſt and concluſive; *& vice verſá.*

IV. That, as there are Objects intirely oppoſite and *diſparata*, ſo there muſt be Faculties in intelligent Creatures, ſuited to thoſe different Objects, differing according to the Diverſity of the Objects.

V. That if Gravitation be the Principle of Activity in Bodies; that of Re-union, with their Origin, muſt, by this analogical Neceſſity, be the Principle of Action in Spirits.

VI. That material Subſtances are the ſame with ſpiritual Subſtances, of the higher Orders, at an infinite Diſtance, or that material Subſtances are ſpiritual Subſtances infinitely condenſed or contracted, ſince in the Scale of Exiſtence, the firſt are ſuppoſed at an infinite Diſtance from the latter.

VII. That there is ſome Analogy between the Conſtitution, Temperaments, and Complexions of ſpiritual Beings, and the known different Textures,

tures, Elements, and Faculties of material Subſtances.

These, I think, as they are neceſſary Conſequences from the preceeding Principle, ſo they are the main Pillars, and ſome of the principal Propoſitions of this ſecond Chapter, which, if underſtood and granted, every Thing elſe will either be eaſily received, or may be ſafely rejected, without any Hazard to the main Syſtem.

After all, ſeeing my whole Intention and Deſign, in advancing and publiſhing theſe Speculations, was to beget in the Minds of Men, noble, generous, and magnificent Sentiments, of God and his Works, that thereby they might be more powerfully engaged, to love, adore, and ſerve Him; to convince them of the Degeneracy and Corruption of the whole Race of Mankind; of the Neceſſity of expanding and cultivating their ſuperior Faculties, by a faithful Obedience to the divine Attraction and Drawings in their Hearts; and thereby, of begetting in their Souls, Charity, or the pure Love of God, and of all his Images in a proper Subordination: All which can by no other Means be brought about, but by a careful copying after, and imitating the Model and Pattern the BLESSED JESUS has ſet us in his Life and Doctrine: I ſay, ſince this was my whole End and Aim, in advancing and publiſhing theſe Speculations; If any Perſon ſhall think fit to controvert them, he may do it very ſafely for me. For being ſatisfied in the Honeſty and Simplicity of my Intentions, and of the Uſe and Benefit theſe Speculations have been to my ſelf for theſe Ends and Purpoſes; I am firmly reſolved, not to ſpend my Time in idle Diſputes. If Others differ with me, about the Truth and Reality of theſe Speculations, or their Uſefulneſs to, and Influence on the Ends propoſed, or are not diſpoſed to reliſh or receive them, they may let them alone or reject them; it is equal to me. All I ſhall be ever prevailed on to do, in ſuch a Caſe (excepting always, in caſe thoſe who either are my lawful Superiors, or whom I look on my ſelf obliged in Conſcience to obey, ſhall command otherwiſe,) ſhall be; to amend, alter, or retract what I ſhall be perſuaded is amiſs in the future Editions of this Work, if it ſhall have any more.

III. The third Chapter is what the reverend and ingenious Mr. *John Craig* ſent me about ſeven Years ago, when I deſired him (being low in my Health, and otherwiſe engaged) to write me down his Thoughts on, correct or alter, what I had formerly publiſhed on this Head in the firſt Edition of this Work, in order to a ſecond Edition. I have altered or added nothing, but one Note before his Additions, and that in Italick Characters.

To conclude, if any Perſon, by either of the Parts of this Work, ſhall be moved to adore, worſhip, or love *the lovely and adorable Author of his Being*, (who is wonderful in all his Works, and great in the leaſt;) I ſay, if any one ſhall be wrought on thereby to love him more, or ſerve him better, I ſhall have the whole Reward of my Labour, having intended it ſolely for his Glory, and the Good of my Fellow Creatures; and having, I hope, in the whole, and each ſingle Part, as far as my Weakneſs and Corruption would permit, diſengaged my ſelf from all ſiniſter Ends, from all Fraud, Malice, Vain-glory and Hypocriſy.

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PART II.

CHAP. I.

Of the Nature and Kinds of Infinites. *Of ſome of their reſpective Qualities. and of a New Arithmetick of* Infinites.

THAT we may reaſon, as cautiouſly as poſſibly we can, about Matters ſo intricate, and ſo far removed from the common Way of Thinking, as the Nature and Qualities of *Infinites*, and the other Subjects of this Chapter are; we ſhall begin with *Definitions* and *Axioms*, and proceed to ſome general *Propoſitions*, demonſtrated after the plaineſt manner, till we obtain Principles to found our conſequent Reaſonings on; and then draw ſuch Corollaries, from the ſeveral Parts, or from the whole, as ariſing neceſſarily from them, may be of uſe to aſcertain ſome Speculations advanced in the foregoing Treatiſe; or may otherwiſe help to conduct the Underſtanding in thoſe other Sciences, where they may find a Place.

Definition I.

QUANTITY is what may be increaſed or diminiſhed.

THOUGH this Definition may not exhauſt the metaphyſical Nature of Quantity; yet it points out that Quality in it, that is here chiefly regarded; for every Quantity may be increaſed or diminiſhed, and that continually, as ſhall be afterwards ſhown.

Definition II.

A BODY is an extended, impenetrable, paſſive, diviſible, unintelligent Subſtance.

THIS Definition alſo, though it exhauſt not the internal Eſſence and intimate Nature of Matter, yet it ſums up its ſenſible and moſt conſtant Qualities, by which it is diſtinguiſhed from every thing elſe.

Defini-

Definition III.

THE inherent Principle of Activity, in the great Bodies of the Univerſe, is Gravitation or ſomething analogous thereto.

THO' I am perfectly convinced, from the Simplicity and Uniformity of the divine Nature, and of all his Works, that there is ſome one great and univerſal Principle, running through the whole Syſtem of Creatures analogically, and congruous to their relative Natures; which is the ſame in all Bodies great and ſmall, and the Origin of all their natural Actions upon one another, with regard to their different Circumſtances; and that there is not a different Principle for the natural Actions of the leſſer Bodies from that which is the Principle of the natural Actions of the greater Bodies of the Univerſe, but one and the ſame Principle in both, acting differently in different Circumſtances: Yet ſince Gravitation, or ſomething analogous thereto, ſeems neceſſary for accounting for the conſtant and regular Motions, and Actions upon one another, of the great Bodies of the Univerſe; Gravitation or ſomething analogous thereto muſt be a neceſſary Conſequence in the greater Bodies of the Univerſe, of this more univerſal Principle, and the Origin of the Activity of Bodies.

Defi-

Definition IV.

A SPIRIT is an extended, penetrable, active, indiviſible, intelligent Subſtance.

Body and *Spirit* are in every other Quality oppoſite, except in *Extenſion*; therefore as the forgoing *Definition* of Body ſumms up its ſenſible and moſt conſtant Qualities, ſo to aſſign the *Definition* of Spirit, there was nothing to be done, but to join the oppoſite Qualities of Body, to that of *Extenſion* or extended Subſtance.

Definition V.

THE *Principle* of *Action* in ſpiritual Subſtances is, or ought to be, that eſſential one of REUNION with the *Origin* of their Being, impreſſed on every individual of this Rank of Creatures.

THE univerſal *Principle* of *Action*, mentioned in the third Definition, that runs through all the *Syſtem* of Creatures, muſt *analogically* be carried through every individual of *ſpiritual* Beings, and can be nothing but this eſſential Principle of REUNION with the *Origin* of their Being, as ſhall be afterwards demonſtrated at large.

Defi-

Definition VI.

A *finite Quantity* is that, of which the Bounds or Limits, beyond which it cannot reach, are *aſſignable.*

THUS a Line is finite, when both its Extremities are given, or the Points which are its Limits, beyond which it cannot reach, are *aſſignable*: An *Area* is finite, when its terminating Lines are *aſſignable*; a *Solid* is finite, when its terminating Planes are *aſſignable*; a *Number* is finite, when the Unities (which are its Limits) of which it conſiſts, or the Bounds beyond which it cannot reach, are *aſſignable.*

Definition VII.

AN *infinite Quantity* (in its ſimpleſt Nature and loweſt Degree) is that, ſome one or more of the Limits or Bounds of which, beyond which it cannot reach, are not *aſſignable.*

THUS a *right Line,* one or both of whoſe Extremities are not *aſſignable,* or the Points beyond which it cannot reach, are not *aſſignable,* is an *infinite right Line.* An *Area,* one or more of whoſe terminating Lines are not *aſſignable,* is an *infinite Area*; a *Solid,* one or more of whoſe terminating Planes are not aſſignable, is an infinite Solid. A Number, increaſing continually, whoſe laſt Increaſe is not aſſignable, or the Bounds, beyond which it cannot reach, are not aſſignable, will at laſt make an infinite Number.

Scholium.

An infinite Number may be ſuppos'd to be generated by the perpetual Addition of a finite Number to it ſelf. Thus $1+1+1+1+1$, *&c.* $a+a+a+a+a$ *&c.* become infinite. Or, it may be ſuppoſed to be generated, by the perpetual Addition of finite Numbers, increaſing in a regular Progreſſion, and in one conſtant Proportion one to another. Thus $1+2+4+8+16$, *&c.* (where the finite Terms perpetually increaſe in the *Ratio* of 1 to 2.) and $1+a+a^2+a^3+a$ *&c.* (where the finite Terms perpetually increaſe in the *Ratio* of 1 to a) become infinite. And it is the ſame in all other infinite Series, regularly generated; or laſtly, the infinite Number may be ſuppoſed to be generated by the perpetual Addition of finite Numbers, in no certain proportion one to another, nor in any regular Progreſſion, ſuch as $7+1+30+5+2+25$ *&c.* Of theſe laſt Kinds of Infinites we have here no Conſideration; for being of no conſtant or regular Nature, but merely caſual and fortuitous, they can afford no *Medium* for reaſoning. The principal Deſign of this Chapter, ſo far as it concerns theſe two Kinds of Infinites, is to find out a *Method* for reſolving the ſecond Kind of Infinites into the firſt, when it is poſſible; in order then to obtain a juſt Notion of theſe Infinites, let us firſt diſtinguiſh Infinite in general, into *relative* or *creaturely* Infinite, and *Supreme* or ABSOLUTE Infinite (of which the firſt is but a created Image or Picture, as will be afterwards ſhewn) let $\odot$ ſtand for Finite in general, and ∞ ſtand for Infinite in general as they reſpect Numbers; then $\odot\ 1$ and $\infty\ 1$ will be finite and infinite (as they reſpect Numbers) of the loweſt Degree and ſimpleſt Nature; Unity being the ſimpleſt Number.

Definition VIII.

Relative Infinite (in its ſimpleſt Nature and loweſt Degree) is an infinite Quantity, as it ſtands related to a given Finite, by the perpetual Addition of which to it ſelf it is generated.

THUS $\infty\ 1$, is a *relative Infinite*, as it ſtands related to 1, a given Finite, by the perpetual Addition of which to it ſelf, it is generated; that is, $\infty\ 1 = 1 + 1 + 1 + 1 + 1$ *&c.* And $\infty\ a$ is a *relative Infinite*, as it ſtands related to the given Finite a, by the perpetual Addition of which to it ſelf it is generated, or $\infty a = a + a + a + a + a$ *&c.*

Scholium.

IN the ſame Relation, that relative Infinite ſtands above a given Finite in *aſcending*, in the ſame may another Quantity be ſuppoſed to ſtand below it in *deſcending*, in which Caſe, we ſhall have a *relative infinitely great* Quantity in *aſcending* above the given Finite; and a *relative infinitely little* Quantity in *deſcending* below it. So that relative Infinite in general may be aptly diſtinguiſhed, in reſpect of the given Finite, into *relative infinitely great*, and *relative infinitely little.* For brevity ſake, we ſhall call the firſt *relative Infinite*, the ſecond *relative Nothing.*

Definition IX.

Relative Nothing (in its ſimpleſt Nature and loweſt Degree) is an *infinitely little* Quantity, as it ſtands related to a given Finite, by the perpetual Subſtraction of which from it ſelf it is generated. Let o ſtand for Relative Nothing.

THUS $o\,1$ is a *relative infinitely little* Quantity, as it ſtands related to Unity, by the perpetual Subſtraction of which from it ſelf, it is generated; that is $o\,1 = 1 - 1 + 1 - 1 + 1 - 1 + 1 - 1$ &c. and $o\,a$ is an infinitely little Quantity, as it ſtands related to the given Finite a, by the perpetual Subſtraction of which from it ſelf, it is generated; that is $o\,a = a - a + a - a + a - a$ &c.

Definition X.

An *indefinite Quantity* (in its ſimpleſt Nature and loweſt Degree) is ſome mean Proportional, between Finite, and relative Infinite or relative Nothing.

For, as in *deſcending* from 1 to 0, we do not immediately ſlip from finite to relative Nothing, but muſt neceſſarily paſs through the intermediate Steps $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$. &c. in Arithmetical Progreſſions; and in *aſcending* from 1 to ∞ we muſt paſs through the Steps between both 2, 3, 4, 5, &c. in the ſame Kind of Progreſſions. So in the *Geometrical* Progreſſions, in *deſcending* from 1 to 0, we muſt paſs through theſe mean Proportionals $\sqrt[2]{0}$ $\sqrt[3]{0}$ $\sqrt[4]{0}$ &c. and in the ſame Progreſſions, in *aſcending* from 1 to ∞, we muſt paſs through the mean Proportionals $\sqrt[2]{\infty}$ $\sqrt[3]{\infty}$ $\sqrt[4]{\infty}$, &c. and theſe we call *Indefinites*. Thus in *Geometry*, if we could imagine a *Circle*, drawn upon the ſumm of a finite right Line repreſenting 1, and an infinite right Line repreſenting $\infty 1$, as a *Diameter*. A Perpendicular erected on the Point where theſe two Lines meet, reaching the *Circumference*, would repreſent the *indefinite* right Line implied by $^2\sqrt{\infty 1}$. Let ∞ ſtand for ſuch *indefinite* Quantities in general.

Scholium.

UNITY being the ſimpleſt *finite* Number, by conſequence $\infty 1 = 1 + 1 + 1 + 1 + 1$ *&c.* muſt be the ſimpleſt *infinitely great* Number. And $01 = 1 - 1 + 1 - 1 + 1 - 1$ *&c.* muſt be the ſimpleſt *infinitely ſmall* Number. And by reaſon the greater the Number denominating the Root is, the leſs the Root it ſelf will be, therefore $^\infty\sqrt{\infty 1}$ ($=$ to any finite Number greater than Unity) will be the ſimpleſt Indefinite. Put $^\infty\sqrt{\infty 1} = y = \infty^{\frac{1}{\infty}}$ therefore $\frac{1}{\infty} L \infty = Ly$. but $L \infty = \infty$ (as will be ſeen by the laſt Chapter) therefore $\frac{1}{\infty} \times \infty = Ly$. that is $\frac{\infty}{\infty} = 1 = Ly$. But 1 may be the *Logarithm* of any finite Number, greater than Unity, in the Scale of Proportionals $1^1.\ a^0.\ a^2.\ a^3.\ a^4.$ *&c.* therefore $^\infty\sqrt{\infty 1}$ may be any finite Number greater than Unity. The *Indefinites* of the firſt Degree, that is wherein the Number denominating the Root is an Integer, may be univerſally thus expreſſed

$$^m\sqrt{} = {}^m\sqrt{\overline{\infty}} = \overline{1 - |\quad}^{\,m} = 1 + \frac{1}{m} + \frac{1 \times \overline{m+1}}{m \times 2m} +$$

$+\frac{1\times\overline{m+1}\times\overline{2m+1}}{m\times 2m\times 3m}+\frac{1\times\overline{m+1}\times\overline{2m+1}\times\overline{3m+1}}{m\times 2m\times 3m\times 4m}+$ &c. Now as $\infty\,1=1+1+1+1+1$ &c. and $0\,1=1-1+1-1+1-1$, &c. and $\infty\sqrt{\infty}\,1$. are of the ſimpleſt Nature, ſo alſo are they of the loweſt Degree. The ſuperior Degrees being generated of $\infty\,1$, after the ſame Manner that $\infty\,1$ is of 1, for if we add $\infty\,1$, perpetually to it ſelf, we ſhall have a *relative infinitely great* Quantity, of the ſimpleſt Nature in its Kind, but of a higher Degree, *viz.* $\infty\,1+\infty\,1+\infty\,1+\infty\,1$ &c. $=1+1+1+1+1+1+1$ &c. $+1+1+1+1+1+1$ &c. $+1+1+1+1+1$ &c. &c. = (Since a perpetual Addition of any Quantity to it ſelf, is equal to a Multiplication by $\infty\,1$) $\infty\,\overline{1\times 1+1+1+1+1+1}$ &c. $=\infty\,1\times\infty\,1=\infty^{2}$. So $\infty a+\infty a+\infty a+\infty a$&c. $=\infty\times\overline{a+a+a+a+a+a}$ &c. $=\infty\times\infty a=\infty^{2}a$. After the ſame manner $0\,1=1-1+1-1+1-1$ &c. If ſubſtracted perpetually from it ſelf, it becomes $0\,1-0\,1+0\,1-0\,1$ &c. $=\overline{1-1+1-1+1-1}$ &c. $-\overline{1-1+1-1+1-1}$ &c. $+\overline{1-1+1-1+1-1}$ &c. $-\overline{1-1+1-1+1-1}$ &c. $+$ &c. = (ſince a perpetual Subſtraction of a Quantity from it ſelf is the ſame with a Diviſion by ∞) $\overline{1-1}$

$\frac{\overline{+1-1+1-1\ \&c.}}{\infty}=\frac{0}{\infty}=\frac{1}{\infty}\times 0=0\times\overline{1-1+1-1+1-1\ \&c.}=0^2$. Thus we have a relative Nothing leſs than 0, but of a ſuperior Degree; for as *relative infinitely great* Numbers increaſe in their Value, by being raiſed to ſuperior Degrees, ſo *relative infinitely little* decreaſe, becauſe the firſt perpetually aſcend from Finite, the latter *deſcend* perpetually further from it. And thus all the Degrees, that finite Quantities admit of, may be formed from relative Infinites and Nothings. And as we have Indefinites ${}^m\surd\infty$ between 1 and Infinite in *aſcending*, and ${}^m\surd 0$ between 1 and 0, in deſcending, ſo *analogous* to their Natures we have the ſuperior Degrees ${}^m\surd\infty^n$ and ${}^m\surd 0^n$. Nature in all theſe Caſes admitting of no Bounds nor Limits.

Definition XI.

Abſolute or SUPREME Infinite, in a proper Senſe, is one, Individual, admitting of neither Increaſe, nor Diminution, or of any Operation that *mathematical* Quantity is ſubjected to.

THIS will be better underſtood afterwards.

Defi-

Definition XII.

Abſolute Nothing in a proper Senſe is neither capable of increaſing nor diminiſhing, nor of any wiſe altering any *Mathematical* Quantity to which it is applied, but ſtands in full oppoſition to *abſolute* Infinite.

Axioms.

I. That which is greater or leſs than any poſſible finite Quantity, how great or little ſoever, muſt be a *relative Infinite, Indefinite,* or *Nothing,* and which of all theſe Three, the given Quantity is, the State of the Caſe will always determine.

BESIDES, *Infinite, Indefinite,* and *Nothing,* relatively conſidered, we have no *Idea*'s of Quantity, and the Definitions of theſe already given, applied to the State of the Caſe under Conſideration, will always determine which of theſe the Quantity aſſigned muſt be.

II. Number being the ſimpleſt Meaſure of Quantity, and a proper Unity being the Meaſure of all Number, a proper Unity is the Meaſure of all Quantity.

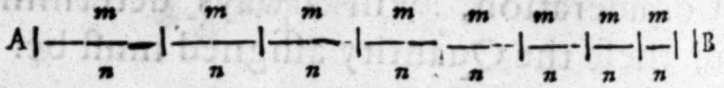
THAT Unity is the Meaſure of all Integers is evident: And in Fractions, the *Denominator* determines the proper Unity, whereof the *Numerator* determines the Number. Wherefore in the following Propoſitions Inſtead of Quantity, we ſhall frequently uſe its Meaſure Number, to expreſs their Meaning and to demonſtrate their Truth.

Propoſitions.

I. *Quantity* may be increaſed or diminiſhed in *Infinitum.*

CASE 1. That *Quantity* may be encreaſed in *Infinitum,* is evident from hence; that ſince it is certain, a finite Quantity may be added to a Finite, what may be once done, may be done again and again, and conſequently may be done any Number of Times, greater than any finite Number how great ſoever, that is by *Definition* 4. and *Axiom* 1. Quantity may be increaſed in *Infinitum.*

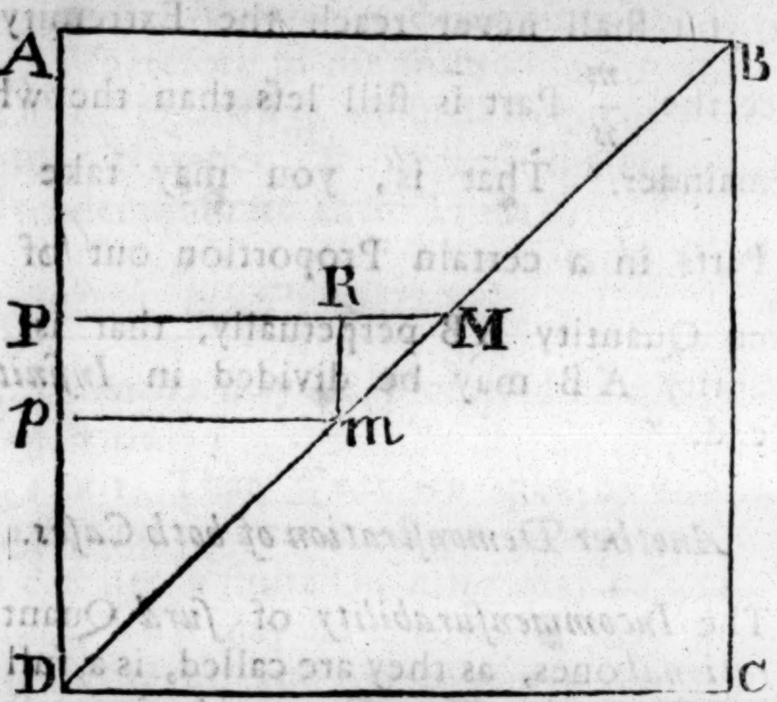
CASE 2. That *Quantity* may be diminiſhed in *Infinitum* is evident from hence, that out of the given Quantity A B, by the *Sixth Book of Euclid*, you may take out any given $\frac{m}{n}$ Part, and out of the Remainder, you may alſo take out any $\frac{m}{n}$ Part, for the ſame Reaſon that you can take it out of the firſt given Quantity A B, and ſo on continually, and



yet you ſhall never reach the Extremity B, ſince the $\frac{m}{n}$ Part is ſtill leſs than the whole Remainder. That is, you may take out $\frac{m}{n}$ Parts in a certain Proportion out of the given Quantity AB perpetually, that is, the Quantity AB may be divided in *Infinitum*. q. e. d.

Another Demonſtration of both Caſes.

The *Incommenſurability* of *ſurd* Quantities to *rational* ones, as they are called, is a full Demonſtration, that *Quantity* may be increaſed or diminiſhed in *Infinitum*.



Let ABCD be a Square, whoſe *Diagonal* is BD, putting the Side AD = 1. Then is AD to DB, as 1 to $\sqrt{1}$. Take, in the Side AD, a Part P*p* the leaſt poſſible, draw PM. *pm.* parallel to AB and R*m* parallel to AD. Since the Triangles BAD and MR*m* are ſimilar, R*m* will ſtill be to *m*M as 1 to $\sqrt{1}$. therefore it is impoſſible to find in AD a Part how ſmall ſoever, that taken, any finite Number of Times, how great ſoever, ſhould be equal to *m*M. So that let AD be divided into Parts how ſmall ſoever, and how many ſoever, yet ſtill DB may be further divided. That is *per Def.* 5. and 4. and *Axiom* 1. DB may be divided in *Infinitum*, and AD increas'd in *infinitum*. q. e. d.

Corol.

Corollary I.

HENCE it is evident, that to aſſign the abſolutely greateſt relative Infinite, or the abſolutely leaſt relative Nothing is a plain Contradiction, ſeeing both theſe are ſtill Mathematical Quantities, (as is plain from Def. 1. 4. and 5, and ſhall be afterwards further demonſtrated;) and ſo by this Propoſition, are capable of further Increaſe or Diminution; and ſo the aſſigned can neither be the greateſt, nor leaſt abſolutely.

Corollary II.

HENCE, and from Def. 4. and 5, we may diſcover wherein the ſpecifick Difference between Finites and relative Infinites or Nothings conſiſts: To wit, in the limited Increaſe or Diminution of the former, and in the Perpetuity of the Increaſe or Diminution of the latter; for as ſoon as the Increaſe or Diminution in theſe latter Stops, they become limited and aſſignable, and conſequently Finite, and thereby, no Part of the deſired Infinite.

Corollary.

HENCE it appears, that an Infinite of either Sort is (as to all Arithmetical Operations on it, with due Regard to the Perpetuity of its Increaſe, or Diminution) of the Nature of an unknown Quantity in *Algebra*. For as in this, from the State of the Problem, we perform Arithmetical Operations on it, as it were known, and thereby we ſometimes do, and ſometimes do not determine its Value, but by Approximation: So on this, we may perform the like Operations as upon an unknown Quantity, with due Regard to its particular Nature, and the State of the Problem, and thereby often diſcover the ſpecifick Genius of its Progreſſion, which is always regular and harmonious, as will be afterwards ſeen.

Propoſition II.

Unity divided by an infinite Number of Unities makes the Quotient relative Nothing or $\frac{1}{\infty\,1} = 0$.

Demonſtrat. $\infty\,1 = 1 + 1 + 1 + 1 + 1$ &c. *per* Def. 4. and $01 = 1 - 1 + 1 - 1$ &c. *per* Def. 5. divide 1, by $1 + 1 + 1 + 1 + 1$ *&c.* and by the common Operations of *Algebra*, you ſhall have $1 + 1 + 1 + 1 + 1 (\div$ &c.$)\ 1 - 1 + 1 - 1 + 1$ &c. $= 0$. q. e. d.

Corol.

Corollary I.

FROM hence it is evident, that Unity divided by relative Nothing is equal to $\infty\ 1$. for $1-1+1-1+1-$ *&c.* $=0)\ 1\ (1+1+1+1+1$ *&c.* $=\infty\ 1$ therefore $\frac{1}{0}=\infty$.

Corollary II.

HENCE alſo it follows, that $\infty \times 0 = 1$. But this may be demonſtrated otherwiſe thus. $\infty\ 1 = 1+1+1+1+1$ *&c. per Def.* 4 Multiply both by *o*, and then it is, $\infty\ 0 = 01+01+01+01$ *&c.* $= 0 \times \underline{\overline{1+1+1+1+1\ \&c.}}$ but by *Prop*, 2. $0=\frac{1}{\infty}$. and $1+1+1+1$ *&c.* $=\infty$. therefore $\infty \times 0 = \frac{1}{\infty} \times \infty = \frac{\infty}{\infty} = 1$ q. e. d.

Propoſition III.

As *Finite* in general is to *relative Infinite* in general, ſo is *relative Nothing* to Unity. That is $\odot : \infty\ \odot :: 0 : 1$.

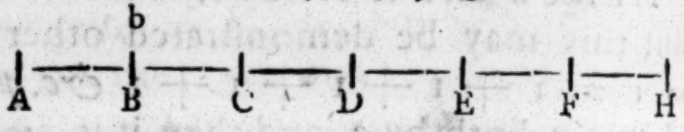
Demonſtrat. By 1. *Corollary Prop.* 2. $\frac{1}{0}=\infty$. Multiply both by $\odot$ then $\frac{\odot\ 1}{0} = \infty\ \odot$ that is $\odot\ 1 = \infty\ \odot\ 0$. therefore $\odot : \infty\ \odot :: 0 : 1$. q. e. d.

Pro.

Propoſition. IV.

Relative Nothing is a real *mathematical* Quantity, and implies the leaſt Part of the Finite, to which it is related or compared.

Demonſtrat. This is evident from the Generation of *relative Nothing*, aſſigned in *Def.* 5. But to demonſtrate the Propoſition, without Regard to this Definition, let AH infinitely produced from A, be divided into equal Parts AB, BC, CD, DE, *&c.* So that an equal Part of this Line may denote any Number. Suppoſing AB $=1$, let x denote any Number; for Example $x = AB$, $y = Ab$, then by the common Rules of Diviſion $\frac{1}{y-x} = \frac{1}{y} + \frac{x}{y^2} + \frac{x^2}{y^3} + \frac{x^3}{y^4}$ &c. Now ſuppoſe b infinitely near to B, then $y - x = Bb\ \dot{x}$ & $\frac{1}{\dot{x}} = \frac{1}{y-x}$ but $x = \dot{x} + \dot{x} + \dot{x} + \dot{x}$ *&c.* by *Def.* 4. that is $x = \dot{x} \times \overline{1 + 1 + 1 + 1}$ *&c.* therefore $\frac{1}{\dot{x}} = \frac{1}{x} + \frac{1}{x} + \frac{1}{x} + \frac{1}{x}$ *&c*, But by Suppoſition A B $= x = 1$ therefore $\frac{1}{\dot{x}} = 1 + 1 + 1 + 1 + 1$ *&c.* $= \infty\ 1$. But by *Corollary* 1, *Prop.* 2. $\frac{1}{0} = \infty$ therefore $\frac{1}{\dot{x}} =$



$\frac{1}{4}$ and

$\frac{1}{o}$ and conſequently $\dot{x} = o$. But $\dot{x}$ being a real Mathematical Quantity, by Def. 1. o alſo muſt be a real Mathematical Quantity, and the leaſt Part of Unity to which it ſtands related or compared. q. e. d.

Corollary I.

SINCE ∞ aſcends from the given Finite in the ſame Manner that o deſcends below it, and ſince o is a real Mathematical Quantity, ſo alſo muſt the ∞ be. And as o is the leaſt relatively below it in its own Order, ſo is ∞ the greateſt relatively above it in its Order, but both below and above theſe, we may deſcend or aſcend in a higher Order or Degree, without Bounds or Limits.

Corollary. II.

WHEN a *Curve* is ſaid to meet with its *Aſymptot*, and when in the common *Hyperbola* we obtain the *Area* $\frac{1}{1-1}$ or in the Example propoſed we put $y = x$, in theſe and ſuch like Caſes we mean only, that in the firſt Caſe the *Ordinate* is infinitely little, in the ſecond, we mean the leaſt Part of the *Abſciſs*, and in the third that y and x muſt differ only by an *infinitely little* Part of x, or $\dot{x}$ only; and not that they are *abſolutely*, equal, elſe there would be no Diviſion, and conſequently no Quotient, as ſhall be afterwards ſhewn.

Corollary III.

SINGE by *Corollary* 1, 2*d Prop.* $\infty = \frac{1}{0} = \frac{1}{1-1+1-1+1}$ &c. $= 1+1+1+1+1$ &c. Since alſo $\frac{1}{1-1} = 1+1+1+1+1$ &c. as is evident from the common Rules of Diviſion, ſeeing the two firſt Terms only effect the Quotient, (all the reſt being but Repetitions of the ſame Terms;) and laſtly, ſeeing the ſame Quotient is obtained whether the Diviſor be $1-1$ or $1-1+1-1+1-1$ &c. the reiterated Multiplication of the Quotient upon the Diviſor, producing the ſame Effect (in the actual Operation of the Diviſion) which ſoever of the Diviſors we chooſe, we may ſafely put in common Caſes $1-1$ or $2-2$ or $a-a$ for o.

Corollary IV.

FROM hence, and *Def*- 4. we may diſcover the true meaning of the Expreſſion, when it is ſaid a *Quantity* is greater than *Infinite*, or one *Infinite* is greater than another. In theſe and the like Expreſſions $\infty\ 1 = 1 + 1$ $+$

$+1+1+1$ *&c.* is always ſuppoſed the common Standard or Meaſure, to which all others are compared. Proper *Unity* being by *Axiom* 2. the common Meaſure of all Quantity, and when a Quantity x is ſaid to be greater than *Infinite*, the meaning is, that it may be an Infinite, greater than $\infty\ 1$. and when $\infty\ 2$ is ſaid to be greater than $\infty\ 1$, it is no more than to ſay $2 < 1$. In all theſe *relative Infinites* admitting of Compariſon, there is ſtill a particular Finite, to which each reſpectively are related, and it is on theſe Finites, that the comparing the *Infinites* among themſelves is founded. Thus

$$\infty\, 2 = \frac{2}{0} = \frac{2}{1-1} = 2+2+2+2 \text{ \&c. when}$$

$$\text{compared with } \infty\, 1 = \frac{1}{0} = \frac{1}{2-1} = 1+1+$$

$1+1$ *&c.* the finite Parts of theſe two Infinites, *viz.* 2, and 1, are the Subjects of the Compariſon. And when $\infty\ 2$ is ſaid to be greater than $\infty\ 1$, it means only that the Finite Parts, of which they are generated, are as 2 to 1, or theſe in the firſt are double of thoſe in the ſecond. It is the ſame thing as in ⊙2, compared with ⊙1, or 2x with 1x, nothing is meant in either, but that 2 is greater than 1 : ⊙, and x, and ∞, being as to this Caſe equally unknown Quantities, which may be thrown out in the Compariſon, and *univerſally*, in all relative Infinites $\infty\, n$, $= n + n + n + n$ &c. and in all relative Nothings $o\, n$, $= n - n + n - n + n - n$ &c. $\infty\, n = \frac{n}{o}$ is the Expreſſion of the *Ratio* of the firſt Series, and $o\, n = \frac{n}{\infty}$ is the Expreſſion of the *Ratio* of the ſecond Series, and theſe two $\frac{n}{o}$ and $\frac{n}{\infty}$ are proper Subjects of the Compariſon, where n may admit of all the Relations that Finites have among themſelves.

Propoſition V.

Relative Infinites, *Indefinites*, and *Nothings* (with the proper Limitations peculiar to each) admit of all the Degrees and *arithmetical* Operations, that finite or *mathematical* Quantities are ſubjected to.

Demonſtrat. This is evident from *Def.* 4, 5, and 6, and the *Scholia* adjoyning to theſe, and is alſo manifeſt from the precedent *Prop.* and its firſt *Corollary*, as to relative Infinites and Nothings; and ſhall be afterwards ſhewn, as to Indefinites; to wit, that all theſe are ſtill *mathematical* Quantities, capable of Increaſe and Diminution *in Infinitum*, and conſequently muſt admit of all theſe *arithmetical* Operations (with proper Limitations peculiar to each) that finite Quantities are ſubjected to. And to confirm this, we may obſerve in Nature a Reſemblance of theſe higher Degrees of *relative Infinites*, and conſequently by *Analogy* of *relative Nothings* alſo. For if *Space* be *infinite*, as ſhall be afterwards demonſtrated, it muſt be ſuppoſed equal to an infinite *Cube* or *Sphere*, whoſe *Diameter* will be as $\infty\ 1$, its Section through this *Diameter* as ∞^2, and its *Content* as ∞^3. q. e. d.

Propoſition VI.

Relative Infinite has to the Finite, with which it is compared, no finite Proportion, or Finite, when compared with its proper relative Infinite, becomes relative Nothing.

Demonſtrat. This is evident from 4 *Corollary Prop.* 4. the *Ratio* of the relative Infinites in general $\infty\, n = n + n + n + n$ &c. being $\frac{n}{o}$ that is $\infty\, n = \frac{n}{o}$ therefore $\infty\, n : n :: 1 . o$. q. e. d. on the other Side let us ſuppoſe the *Ratio* of the relative Infinite in general $\infty\, n$ to n, to be a finite *Ratio* $\frac{s}{r}$ then $\infty\, n = \frac{r}{s}$ which is impoſſible by 2 *Corollary*

rollary Prop. 3. wherefore ſince $\infty\, n : n :: 1 : 0$ by compoſition of Ratio's, $\infty\, n + n : n :: 1 + 0 : 0$ but $1 + 0$ is but 1. therefore $\infty\, n + n$ is but $\infty\, n$, or n a finite Quantity when compared with its relative Infinite $\infty\, n$ becomes 0. q. e. d.

Corollary I.

Hence, *relative Infinite* in general is to *Finite* in general, as *Unity* is to *relative Nothing:* or any *Finite,* when compared with *relative Infinite* in general, becomes *relative Nothing* : that is $\infty : 0 :: 1 : o$.

Demonſtrat. By the *precedent Propoſition* $\infty\, n : n :: 1 : o :: a : o a$, and by converſion of Ratio's $\infty\, n : a :: n : o a$. Suppoſing then a to be a *finite* Quantity, Nothing but an infinite Number of relative Nothings or o's being equal to 1, by *Corollary* 2. *Prop.* 2. $o a$ muſt be ſtill o, or $a - a + a - a + a - a$ &c. $= o$, by *Corollary* 3, *Prop.* 4. wherefore $\infty\, n : a :: n : o$, and by compoſition of *Ratio*'s $\infty\, n + a : a :: n + o : o$, put $n = 1$ and $a = \odot$ then $\infty + \odot : \odot :: 1 + o : o$. and ſince $1 + o$ is but 1 therefore $\infty + \odot$ is but ∞; wherefore $\infty : \odot :: 1 : o$, or when any Finite is compared by Addition, (or by Subſtraction by Diviſion of Ratio's) with *relative* Infinite in general, it becomes *relative Nothing* q. e. d. The true *Analogy* is this, $\infty\, n$ or $n + n + n +$ &c. $a :: n : a - a + a - a +$

$a+a-a$ *&c.* but becauſe a is ſuppoſed Finite, ao is the ſame (in Caſes of Addition and Subſtraction) with o. But were $a=\infty\ 1$, then $a-a+a-a+a-a$ *&c.* would be $\infty-\infty+\infty-\infty+\infty-\infty$ *&c.* $=\infty\times\overline{1-1+1-1+1-1\ \&c.}=\infty\times o=1$, and $\infty n-\overline{\infty n+\infty n-\infty n\ \&c.}=\frac{\infty}{\infty}n=n$.

Corollary II.

Put n equal to any Integer, then $\infty^{n.1}:n::1:o$, but $o^{n}:o^{n-1}::o:1$, for the firſt being reduced, becomes $\infty:1::1:o$, and the ſecond being reduced, becomes $o:1::o:1$; and therefore when ∞^{n-1} is to be added to, or ſubſtracted from ∞^{n}, it becomes o; and when o^{n} is to be added to, or ſubſtracted from o^{n-1} it becomes alſo o, by *Scholium Def.* 6. and the Caſe is the ſame when the inferior Powers ſuppoſe ∞^{n-2}, or ∞^{n-3} *&c.* are to be added to, or ſubtracted from ∞^{n}, or when o^{n} is to be added to, or ſubtracted from the inferior Powers o^{n-2} or o^{n-3} *&c.* in all ſuch Caſes it is evident from the precedent *Prop.* and its 1. *Corollary*, that ∞^{n-2} ∞^{n-3} and o^{n} become o.

Scholium.

FROM theſe *Propoſitions* and *Corollaries*, an *Arithmetick* of Infinites may be drawn out, different from any hitherto publiſhed, of no contemptible Uſe, or narrow Extent in *Algebra*, and *Geometry*; as will be in ſome Meaſure ſhewn by the third Chapter.

* * *

The Arithmetick of Infinites.

ADdition. $2\infty + 3\infty = 5\infty$. $a\infty + 2a\infty = 3a\infty$. $\infty^2 + 2\infty^2 = 3\infty 2$. $\infty^n + p\infty^n = \overline{1 + p}\,\infty^n$. $a\infty + e\infty = \overline{a + e}\,\infty$. $3\infty + 10 = 3\infty$. $a\infty + a = a\infty$. $\infty^2 + \infty = \infty^2$. $\infty^n + \infty^{n-} = \infty^n$. *Let q be leſs than n, by a finite Integer, then* $\infty^n + \infty^{n-q} = \infty^n$.

Subſtraction. $5\infty - 2\infty = 3\infty$. $10a\infty - 3a\infty = 7a\infty$. $6n\infty - 4n\infty = 2n\infty$. $5\infty - 7\infty = -2\infty$. $7\infty^3 - 5\infty^3 = 2\infty 3$. $9\infty^p - 5\infty^p = 4\infty^p$. $5\infty^p - a\infty^p = \overline{5 - a}\,\infty^p$. $3\infty - 7 = 3\infty$. $a\infty - a = a\infty$. ${}^{p-1}$ $\infty^p - \infty^{p-1} = \infty^p$. *Let q be an Integer, leſs than p, then* $\infty^p - \infty^{p-q} = \infty p$.

Multiplication. $\infty \times 3 = 3\infty$. $\infty \times a = a\infty$. $\infty \times \infty = \infty_2$: $\infty^2 \times \infty 3 = \infty 5$. $\infty^p \times \infty^q = \infty^{p-q}$. $a\infty \times e\infty = ae\infty^2$. 3∞

× *a*

$a \times 4\infty x = 12\infty^2 ax.$ $-2\infty \times -3\infty = 6\infty.$ $\infty a \times +\infty = -\infty^2 a.$

Diviſion. $\frac{\infty}{2} = \frac{1}{2}\infty.$ or $\infty\frac{1}{2}.$ $\frac{\infty a}{e} = \infty\frac{a}{e}\frac{2\infty}{\infty} = 2.$ $\frac{3\infty}{4\infty} = \frac{3}{4}.$ $\frac{\infty a}{\infty e} = \frac{a}{e}\frac{\infty 2}{\infty} = \infty.$ $\frac{\infty^3}{\infty} = \infty^2.$ $\frac{\infty^p}{\infty^q} = \infty^{p-q}.$ $\frac{+3\infty}{-\infty} = -3.$ $\frac{-4}{-2}\frac{\infty}{\infty} = +2.$ $\frac{-\infty a}{+\infty e} = -\frac{a}{e}.$

Fractions. $\infty\frac{2}{3} + \infty\frac{4}{5} = \infty\frac{22}{15}.$ $\infty\frac{a}{e} + \infty\frac{p}{q} = \infty\frac{aq+ep}{eq}$ $-\infty\frac{3}{2} + \infty\frac{4}{3} = \frac{2}{15}\infty.$ $\infty\frac{a}{e} - \infty\frac{p}{q} = \infty\frac{aq-ep}{ep}$ $\infty\frac{a}{e} \times \infty\frac{p}{q.} = \infty^2\frac{ap}{eq.}$ $\infty\frac{a}{e}\Big)\infty\frac{p}{q}\Big(=\frac{ep}{qa.}$ $\infty\frac{5}{4}\Big)\infty\frac{2}{3}\Big(=\frac{8}{15.}$

Thoſe who are ever ſo little acquainted with the *ſpecious Arithmetick*, will eaſily underſtand the Reaſon and Truth of theſe Operations.

Propoſition VII.

Indefinite Quantities are not properly either Finite or Infinite, but between both.

Demonſtrat. An indefinite Quantity is ſome mean Proportional, between finite and relative Infinite, *per Def*. 6. that is, one of thoſe Infinites will be $\infty = \sqrt{\infty}\ \odot$ put $\odot = 1$, and then $\sqrt{\infty}$, will be an indefinite Number. Now this $\sqrt{\infty}$ can neither be Finite (and this Manner of Reaſoning will hold good of any other Indefinite whatſoever) nor Indefinite; not Finite, elſe $\odot^2$ would be Infinite, which is impoſſible; nor Infinite, for the leaſt poſſible Infinite muſt be Infinite in general, divided by the greateſt poſſible Finite x, and then if $\sqrt{\infty}$ be infinite, $\sqrt{\infty} = \frac{\infty}{x}$ and $x^2 = \frac{\infty^2}{\infty} = \infty$. which is alſo abſurd. Again if $\sqrt{\infty}$ be infinite, then is $\infty \times \frac{1}{\sqrt{\infty}}\ (= \sqrt{\infty}) =$ infinite, and $\frac{1}{\sqrt{\infty}} =$ to its relative infinitely ſmall Part, or its relative Nothing $= 0$, and ſo $\infty \times 0 = \infty \times \frac{1}{\sqrt{\infty}} = (\sqrt{\infty})$ to Infinite. But by *Corollary* 2. *Prop*. 2. $\infty \times 0 = 1$, and inſtead of $\infty \times 0$, putting its value 1. in this laſt Equation, it will be $\infty \times 0 = 1 = \infty \times \frac{1}{\sqrt{\infty}} =$ *Infinite*, by Suppoſition; and this ſuppoſed Infinite would become 1, which is abſurd. q. e. d.

Scho-

Scholium.

TAKING the Inſtance of the indefinite Quantity propoſed $\sqrt{\infty}\;1 = \sqrt{1+1+1+1}$, *&c.* It is plain, the Root of any given Number grows greater or leſs, as the Number expreſſing or denominating the Root is leſs or greater; and in $\sqrt{\infty}\;1$, its infinitely little Root or $^{\infty}\sqrt{\infty}\;1$ may be any finite Number greater than Unity, as has been already ſhewn; and its infinitely great Root or $^{1}\sqrt{\infty}\;1$ is infinite. And between theſe lie all the Indefinites that can be formed on 1 and ∞; to wit $^{2}\sqrt{\infty}$, $^{3}\sqrt{\infty}$, $^{4}\sqrt{\infty}$, *&c.* and none of theſe can be properly called either Finite or Infinite, but are in a perpetual Gradation towards either of theſe Extremes, as the Number that denominates the Root grows greater or leſs; and they never become actually finite, but when the Number denominating the Root is actually infinite, nor actually infinite, but when the Number denominating is Unity. And between theſe two *Limits*, they are neither actually finite nor infinite. Next to $^{1}\sqrt{\infty} = \infty$ (in order of the ſimpleſt Indefinites,) is $^{2}\sqrt{\infty} - 1 + \frac{1}{2} + \frac{3}{\infty} + \frac{5}{16} + \frac{35}{128}$ *&c.* wherein (by the quick Increaſe and Greatneſs of the ſucceeding Terms) the laſt becomes $\frac{1}{\infty}$ in a few Number, ſo to ſpeak, of theſe

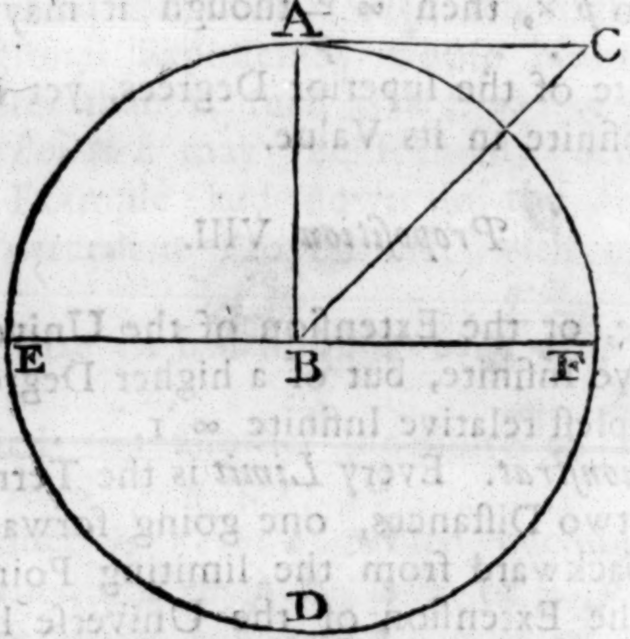
Terms, where the Series terminates, and ſo their Sum $\sqrt[2]{\infty}$. becomes thereby leſs than ∞ 1. as will be more fully explained in the following Chapter, and in $\sqrt[5]{\infty}$ (for example) $= 1 + \frac{1}{5} + \frac{6}{50} + \frac{66}{750}$ *&c.* we ſee the Terms of the Series converge yet faſter, and will thereby terminate at $\frac{1}{\infty}$, much ſooner,. And ſo their Sum $\sqrt[5]{\infty}$ will be much leſs than $\sqrt[2]{\infty}$. And for theſe (even when they riſe to fractional Exponents, whoſe Numerators are greater than 1, ſuch as $\infty^{\frac{2}{3}}$. $\infty^{\frac{3}{4}}$. $\infty^{\frac{4}{5}}$. *&c.*) an *Arithmetick* may be formed, according to the Example laid down in the *Scholium* of the precedent Propoſition, with this addition, that an Indefinite as $\infty^{\frac{q}{p}}$ or $p\surd \quad q$ multiplied by another Indefinite $\infty^{\frac{r}{s}}$ gives the Product $\infty^{\frac{q}{r}+\frac{r}{s}}$. which becomes Infinite, when $q\,s + p\,r$ is $=$ or $>$ $p\,s$ and Indefinite when $q\,s - p\,r$ is $=$ or $>$ $p\,s$. And if an Indefinite as $\frac{q}{\infty}\,r$ be divided by an Indefinite, as $\infty^{\frac{r}{s}}$ the Quotient $\infty^{\frac{q}{p}-\frac{r}{s}}$ is Infinite, when $q\,s - p\,r$ is $=$ or $>$ $p\,s$, but is Finite when $q\,s - p\,r = o$ and only Indefinite, when $q\,s \mp p\,r > p\,s$. as it ought to be according to the Arithmetick of Surds.

N. B. That in expreſſing an Indefinite by $\infty\frac{q}{p}$ it is always ſuppoſed that the Number q is leſs than p, for if q be either greater or equal to p ×, then $\infty\frac{q}{p}$ though it may be an Indefinite of the ſuperior Degrees, yet it is always infinite in its Value.

Propoſition VIII.

Space, or the Extenſion of the Univerſe, is a relative Infinite, but of a higher Degree than the ſimpleſt relative Infinite ∞ 1.

Demonſtrat. Every *Limit* is the Termination of two Diſtances, one going forward, the other backward from the limiting Point, and were the Extenſion of the Univerſe limited, theſe *Limits* would terminate a Space, beyond theſe *Limits*, as well as within them. That is, either the univerſal *Space* muſt be unlimited, and conſequently infinite, or there muſt be *Space* beyond the Limits of univerſal Space, which is abſurd. Again, if the Extenſion of the Univerſe were limited any way, ſo as to become finite, then a *Sphere* of a finite *Diameter* might be found equal to it. For the *Cube* of a finite Side may be found equal to any finite *Content* whatſoever, as is well known, and the *Radius* of a *Sphere* equal to this *Cube* is the *Product* of the *Side* of the *Cube*, multiplied into the *cube Root of* $\frac{3}{2}$ *Parts of the Ratio of the Radius to the Circumference.* Let us ſuppoſe the whole finite Extenſion of the Univerſe equal to the *Sphere*, whoſe Radius is



AB. Let this *Sphere* be cut by a *Plane* through its *Center*, and the *Section* be the *Circle* ADFE, it is certain from the *Elements of Euclid*, that to any given Point A, a *Tangent* AC may be drawn, of which only the Point A falls upon the *Circle*; the reſt of the Line AC falling without it. From whence it is evident, that there muſt be Extenſion without this Circle, or the *Sphere* by the Section of which it is generated. Since a *Plane* paſſing through this *Tangent*, and perpendicular to the *Plane* of this *Circle*, will only touch the *Sphere* in a Point; by which there will be an extended Diſtance remaining between the *Circumference* of the *Sphere*, and the *touching Plane*, in all their Points excepting that one at A; And ſince this is true of every *aſſignable* Extenſion how great ſoever, leſs than Infinite; it is evident the Extenſion of the Univerſe is greater than any *aſſignable* Extenſion, how great ſoever, that is, by *Axiom* 1. it is *relatively Infinite*, which is ſtill more evident from *Def.* 4. Since its Parts are finite and their Sum only infinite, and ſeeing this Infinite is of three Dimenſions, or as $\infty\ 1$ in Height, Breadth, and Depth, therefore it muſt in its Content be as $\infty\ 3$, that is, of a ſuperior Degree to ∞^{1}. q. e. d.

* * *

CHAP.

CHAP. II.

Of the PHILOSOPHICAL *Principles of Revealed Religion.*

Lemma I.

THERE is in all the Works of Nature a *Symmetry* and *Harmony*, running on in a perpetual *Analogy* (with proper Limitations ariſing from the different Circumſtances of the ſeveral Parts) through the Whole and the Parts; or there is a *regular Connexion*, and *uniform Proportion* between ſimilar Cauſes and Effects, a Congruity between the End and the Means. An Aptitude between the Faculty and its Acts, and between the Organs and their intended Uſes in the Whole, and in the ſeveral Parts of this *Syſtem* of Things.

Demonſtrat. This is evident from innumerable Inſtances already diſcovered and aſcertained. And every new Diſcovery in the moſt minute Part in the Works of Nature carries along with it a freſh *Demonſtration* of this Propoſition; one muſt be intirely ignorant of *Philoſophy* and *Mathematicks*, to want a Cloud of Witneſſes to this Truth. For Inſtance, the *ſeſquialter Proportion*, of the periodical Times of the Revolutions of the *primary Planets* about the *Sun*; and of the *ſecundary Planets* about the *primary Ones*; to their *middle Diſtances* from the *Sun* and *primary Planets* reſpectively, obtains univerſally. Their *Magnitudes, Gravities, Denſities*, and their *Velocities* in their *Orbits*, in reſpect to, and about the *Sun* and the *primary Planets*, in the *Planets, Comets*, and *Satellites*, are in a regular and comely Proportion; the ſame *Gravity*, the ſame Law thereof, and the ſimilar Effects of both, obtains through the whole material *Syſtem* of Things. The *Reflexions, Inflexions*, and *Refractions* of Light, are the ſame in all the *planetary* and *cometary* Bodies and Regions; as they are on our terreſtrial Globe, with due Regard to the different *Denſities* of the *Mediums*. The *Circulation* of the *Fluids*, the *Manner* and *Organs* of *Reſpiration* and *Generation*, are *analogically* the ſame in *Man, Brutes*, and *Vegetables*; with proper Limitations ariſing from the differing Circumſtances of theſe Gradations of Animals. The *general Laws of Fluids, Elaſticity* and *Gravity*, obtain in *animal* and *inanimate Tubes*, but ſo far as they are altered in the firſt by *collateral* Cauſes. The whole of *Philoſophy* and *Mathematicks* is nothing but particular Inſtances of this beautiful *Analogy*, and the preceeding Chapters of the *firſt Part* contain nothing but particular Inſtances thereof. And if we deſcend into the *ſpiritual World*, we ſhall find this beautiful *Analogy* preſerved, as far as the different Circumſtances of theſe Beings will permit. And if in this Demonſtration, it were allowed to take in the Suppoſition of a *Being infinitely perfect*, who contrived and executed the whole, and the ſeveral Parts of this *Syſtem* of Things; it is impoſſible it ſhould be otherwiſe: A Being infinitely Wiſe, Simple, and One, muſt neceſſarily bring about ſimilar Ends by ſimilar Means, and perform all his Works the plaineſt, moſt ſimple and ſhorteſt Way poſſible; due Regard being had to the Whole, and the different Circumſtances of the ſeveral Parts. *Wiſdom in Things*, is their *Symmetry*, *Regularity*, and *Aptitude* for obtaining their deſigned Ends and Purpoſes. The *Wiſdom* of a *Machin* conſiſts in the due proportioning of the ſeveral Parts to one another, and to the Whole, for obtaining its propoſed End. *Diſproportion*, *Irregularity*, *Diſcord*, and the having no View or Deſign, are the ſureſt Proofs, and Indications of Chance, Impotence and Folly: A wiſe Man performs all his Works in Number, Weight and Meaſure, and ſure *Infinite Wiſdom*, *Simplicity*, and *Unity*, muſt accompliſh all its Works, with the moſt *conſummate Harmony*, *Proportion*, and *Regularity*. And this in the following

Parts of this Treatiſe, for Brevity's ſake we ſhall call the ANALOGY OF THINGS.

Lemma II.

THIS ANALOGY OF THINGS duly inſtituted, is as certain a *Demonſtration* of the *Exiſtence* and *Wiſdom* of the Author of theſe Things, and of the Contriver of this *Analogy*, as alſo of the true Nature and Qualities of theſe Things diſcovered by this *Analogy*, as any *Mathematical Demonſtration* is of the Propoſition propoſed.

Demonſtrat. No Effect can be, without its proper Cauſe; a wiſe and regular Effect muſt be produced by a wiſe and intelligent Cauſe, and an *infinitely wiſe*, and *infinitely complicated* Effect, muſt neceſſarily imply an *infinitely wiſe* and *Omniſcient* Cauſe. Theſe are ſo certain and infallible *Axioms*, that I know not if in all the Compaſs of human Knowledge, any others come up to the ſame Degree of Evidence: And he can be no proper Subject of *Philoſophy* or *Mathematicks*, who could ſeriouſly deny them, ſince the whole Evidence of both theſe Sciences ſuppoſes the firſt of the *Axioms*, and the reſt are but like multiplying both Sides of the ſame *Equation* by the ſame Terms. I have already obſerved, that the *Wiſdom* of an *Effect*, or Syſtem of Effects, conſiſts in the Proportion or *Analogy* of the ſeveral Parts to the Whole, and to each other, and of the Whole, to the End propoſed; and that Irregularity and no Proportion is the ſureſt Evidence of Want of Contrivance, Wiſdom, and Deſign. Now ſince the ANALOGY OF THINGS, juſt now demonſtrated to be found in all the Works of Nature, in the Whole, in every the moſt minute Part: And in theſe Inſtances of this *Analogy*, without Number, and without End: (Every new Step in the Knowledge of Nature diſcovering freſh Inſtances of this *Analogy*) all theſe, I ſay, do neceſſarily infer an *Infinity of infinitely wiſe* Effects; and therefore theſe Effects muſt as neceſſarily infer the Exiſtence of the Author of theſe Effects, and the Wiſdom of the Contriver of this *Analogy*, as an Effect infers its Cauſe; that is, as certainly as any *Mathematical Demonſtration* infers its Propoſition; ſince its Certainty depends on the Connexion between Cauſes and Effects, and the Truth of this *Analogy* in general. I ſay in the next Place, that the true Nature and Qualities of theſe Effects or Things, diſcovered by this *Analogy*, duly inſtituted, may be as certainly concluded from this *Analogy*, as any *Mathematical Demonſtration* concludes its Propoſition. *Mathematicks* and *Philoſophy*, ſo far as they are juſt and genuine, are but Branches of this *Analogy*. *Mathematicks* are but this *Analogy* applied to *Figures* and *Numbers*.

Philoſophy, properly ſo called, is but this *Analogy* applied to Bodies, or *Syſtems* of theſe, or to the abſtracted Natures of Things; but ſuppoſe the Truth and Neceſſity of this *Analogy*, without which they are but *Jargon* and *Romance*. An Inſtance or two will make the whole Matter clear. Suppoſe it were required to find the *refracted Ray*, when the *refractive Powers* of the two *Mediums*, with the Inclination of the *incident Ray* on the *incident Plane*, are given. Let us borrow a *Corollary* from this *Analogy* of Things, to wit, That the Diſtance between any fixed Point in the *Incident*, and another in the *refracted Ray*, (the *refractive* Powers of the two *Mediums* being regarded,) is the ſhorteſt poſſible; and with this *Corollary* make an exact Computation; we ſhall then find the *refracted Ray* preciſely the ſame with that found out, from other different Principles and Methods (ſuppoſe of *Trigonometry*) where this *Corollary* has had no Place, as the *Geometers* have ſhown. This *phyſical Demonſtration* of this particular Property of Light, (to wit, that in all *Incidences*, the *Sine* of the *Angle* of *Incidence*, is to that of *Refraction*, in a conſtant *Ratio*) ariſing from this *Analogy*, is as certain a Proof of the Exiſtence and Wiſdom of the Contriver of this *Analogy*, as alſo of the true Nature of Light, (as to this particular Property) as any *Mathematical* Demonſtration (ſuppoſe that by *Trigono-*

gonometry) is of the true Nature of the *incident* and *refracted Ray*. For who but an *Omniſcient Artificer* could contrive Light ſo, That among all the infinitely different poſſible Ways, between two aſſigned Points, it ſhould ſingle out that one, which is the ſhorteſt? And it is certain, that the *refracted Ray* is as truly found out by Virtue of this *Analogy*, as by any other more *geometrical* Method. Another Inſtance may be taken from the regular and harmonious Progreſſions of *infinite Series*'s; for Example, in the Powers of the *Binonimal Root*, $\overline{a \quad y}|^{n} = a^{n} + n\,a_{n-1}y + n \times \frac{n-1}{2}\, a^{-n2}y^{2} + n \times \frac{n-1}{2} \times \frac{n-2}{3}\, a^{n-3}\,3\,y$ *&c.* or the *Series* produced by Diviſion $\frac{1}{1-n} = 1 + n^{2} + n^{3} + n^{4}$ *&c.* In a Word, every particular *Problem* in *Algebra* and *Geometry* might be brought as Inſtances of this beautiful *Analogy* of Things; and thoſe who are converſant in the more abſtruſe Speculations of *abſtract Geometry*, can furniſh themſelves with Inſtances ſo ſurpriſing and extraordinary, of the Conſtancy of Nature in this beautiful Harmony, and comely Proportion, even in the largeſt Computations and moſt complex Conſtructions, as far ſurpaſs theſe Inſtances I have brought. Now what but an infinitely wiſe Being could have conſtituted the *intellectual Species* of Things ſo admirably? That all the Terms of theſe Progreſſions ſhould thus go on in ſuch regular and harmonious Proportions, that every ſucceeding Term ſhould be made up of the preceeding ones, modified after one conſtant Way, that by the Caſt of an Eye, the ſaid ſucceeding Term ſhould be aſſigned? Whereas they might have been ordered, other infinitely different Ways, ſo as to have afforded no regular Progreſſion. And does not this *Analogy* and harmonious Progreſſion of theſe *Series's* as certainly give the ſucceeding Terms, as the actual Operations of *Algebra* do? Many more, and yet infinitely more ſurprizing Inſtances of this beautiful *Analogy*, and of the Inferences drawn from thoſe aſſigned, might be given. But theſe may ſuffice for an Illuſtration of this *Lemma*.

Lemma III.

THE Rules which ſeem naturally to ariſe, for the due Inſtitution of this *Analogy* of Things, may be reduced to theſe three. 1. That the Quality, Property, or Idea, on which the *Analogy* is inſtituted, be as ſimple an one, as poſſibly may be; and intirely the ſame, both in the known and unknown Subjects of the *Analogy*. Thus when we reaſon from the Nature and Properties of *Light* and *Gravity* on our *Earth*, to the Nature of *Light* and *Gravity* in the *Planets* and *Comets*; we muſt ſeparate theſe into their moſt ſimple *Ideas* and *Properties* (to avoid Confuſion) as much as may be, and inſtitute an *Analogy* for each, to diſcover, if the *Analogy* hold good in the Complex, and preciſely keep to the ſame *Properties* in the *Planets* and *Comets*, the unknown Subject of the *Analogy*, as were ſuppoſed in the *Earth*, the known Subject of the *Analogy*. Elſe we ſhall run into Confuſion and *Paralogiſm*. 2. The neceſſary Limitations ariſing from the different Circumſtances of the two Subjects of the *Analogy*, as far as they may be known, muſt be cautiouſly and carefully taken into the State of the Queſtion. Thus in the firſt Inſtance, aſſigned in the ſecond *Lemma*, if the different *refractive Powers* of the two *Mediums* had not been preciſely entered into the *Equation*, the Concluſion muſt have proved different from that found out by other Methods. Thus alſo in reaſoning from the Manner of *Generation* in *Animals* to that of *Vegetables*: If the Limitations ariſing from the different Circumſtances and Orders of Being, in theſe two Subjects of the *Analogy*, be not carefully taken into the Reaſoning, the Concluſions muſt prove falſe and erroneous. 3. Both the Subjects of the *Analogy* muſt be known and examined into, as far as may be, in regard to the other Qualities different from thoſe, the *Analogy* is inſtituted upon: But eſpecially the known Subject of the *Analogy* muſt be as fully known as is poſſible, in regard to thoſe Properties on which the Analogy is inſtituted. Thus the more fully we underſtand the Nature and Qualities of our Globe, eſpecially as to *Light* and *Gravity*, and of the *Comets* and *Planets* as to their other Qualities, the more full and perfect ſhall our Concluſions be in regard to theſe aſſigned Qualities, in the *Planets* and *Comets:* The more fully we underſtand the Circulation of the Fluids in *Animals*, the more aptly ſhall we apply them to *Vegetables*. Theſe Rules duly obſerved will render the Concluſions drawn by Virtue of this *Analogy of Things*, as certain as any *mathematical* Demonſtration whatſoever.

Propoſition IX.

THE *viſible*, *intellectual*, and *created Species* of Things, are Pictures, Images, and Repreſentations of the *inviſible*, *archetypal*, and *increated Species* of Things in the Mind of the ſupreme Being.

Demonſtrat. This is evident from *Lemma* 1 and 2. For ſince it is certain, that there are to be found in all the Works of Nature a beautiful Harmony, a comely Proportion, and an exact Symmetry running through the Whole: And ſince this *Analogy of Things*, neceſſarily infers the Exiſtence of the Author of theſe

Things, and the Wiſdom of the Contriver of this *Analogy:* Theſe Things and this *Analogy* could come from nothing elſe but from their original *Ideas* and *archetypal Patterns*, in the *divine Mind* or *Imagination*, and their Harmony and Proportion can poſſibly ariſe from nothing but their being Repreſentations of his *Idea's*, who is *Omniſcient*, and does every Thing in *Number*, *Weight*, and *Meaſure.* There being no other poſſible Way they could be contrived; and he being *Supreme* and *One*, could find nothing without himſelf that they ſhould repreſent. Beſides, it is abſolutely impoſſible, that infinite Power and Perfection ſhould bring any Thing into Being, that had not his own *Signature*, Stamp, or Image on it, for there could be nothing beſides himſelf, whoſe Images they ſhould be; and it is abſurd to imagine they ſhould repreſent nothing at all. Therefore of neceſſity they muſt be Pictures, Images, and Repreſentations of their *Idea's* and original Paterns in the Mind of the ſupreme Being. q. e. d.

Corollary I.

HENCE it is evident that the *viſible*, *intellectual*, and *created Species* of Things, are Images, Pictures, and Repreſentations of the *divine Attributes*, more or leſs perfect, according to their Order in the Rank of Beings.

ings. For ſince they are Images of the *archetypal Idea's* in the *divine Mind*, or *Imagination*, and ſince there was nothing without him to beget theſe *Idea's*, and nothing but his own infinite Perfections, that could repreſent themſelves to him, in order to make theſe *Idea's* ariſe in the Mind: Therefore of neceſſity, they muſt repreſent his *Attributes* or *Perfections*, the only Thing he could have to copy out.

Corollary. II.

HENCE it is evident, that with proper Limitations ariſing from the *infinitely infinite* Diſtance; (ſo to ſpeak) between Finite and *abſolutely Infinite*, we may reaſon analogically from the Nature and Attribute of the *ſupreme intelligent Being*, to the Nature and Properties of *finite intelligent Beings*; and from theſe to thoſe; to wit, by ſuppoſing theſe laſt Images, Pictures, and Repreſentations, at an infinite Diſtance, of the firſt.

Propoſition X.

A *Mathematical* Point, and infinite Space are the two *Limits* of natural or created Things, as to Quantity or Extenſion, in Aſcending or Deſcending from Finite. And neither of them is any Part or Multiple of the other.

Demonſtrat. This is evident from *Prop.* 8. for natural or material Things can have no more Dimenſions than three, and therefore the biggeſt Extreme in created Things, can riſe no further by the *Analogy* of Things than to ∞ raiſed to the third Dimenſion. And ſince infinite Space is as ∞^3, therefore infinite Space is the biggeſt Extreme, or Limit of natural Things aſcending from Finite. And ſince a *Mathematical* Point is the Surface (ſo to ſpeak) of the Extremity of a *Mathematical* Line, which has but one Dimenſion, it muſt of Neceſſity be the leaſt Extreme in deſcending from Finite: In a Word, we can riſe no higher in natural and material Things than infinite Space, ſince that is the *Locus Univerſalis* of all created Beings. And we can deſcend no lower than a *Mathematical* Point, ſince every Thing below it (if poſſible) could have no Place at all, or would be no where, that is, would be abſolute Nothing. And neither of them can be any Part or Multiple of the other, ſince no Number of Points, no not even an infinite Number can make any real natural Quantity, not ſo much as a *Mathematical* Line, therefore, *&c.* q. e. d.

Corollary. I.

HENCE a *Mathematical Point* and *univerſal Space*, are true and genuine Oppoſites in

Nature, and in the *Analogy* of Things, and between theſe two lie all created finite Subſiſtences. No natural Thing can be bigger than *infinite Space*, and no natural Thing can be leſs than a *Mathematical* Point; they differ the moſt widely that natural Things poſſibly can, and have nothing common but *Entity*, and ſo are true and genuine Oppoſites.

Corollary II.

HENCE in the *Analogy* of Things, *Matter* cannot be infinite, in any Senſe of Infinite aſcending from Finite. For ſince *Matter* is of thoſe natural Things, that neceſſarily require all the three Dimenſions, were it infinite, by the *Analogy* of Things, it would neceſſarily be as ∞^3, that is, it would be equal to infinite Space; but ſince the Neceſſity of a *Vacuum* has been demonſtrated in the firſt Chapter of the firſt Part, it cannot be equal to infinite Space, and ſince it cannot be infinite any wiſe but as infinite Space, by the *Analogy* of Things. Therefore *Matter* cannot be infinite, in any Senſe of Infinite aſcending from Finite.

Propoſition X.

Abſolute Nothing, upon a real Quantity produces no Effect at all. Or *abſolute Nothing* can be ſuſceptible of none of theſe *Arithmetical* Applications, to which real Quantities are ſubjected.

Demonſtrat. Suppoſe abſolute Nothing upon a real Quantity could produce any Effect. Let the real Quantity be a, and the Effect m, then abſolute Nothing × $a = m$ that is, abſolute Nothing is equal to $\frac{m}{a}$, that is, abſolute Nothing is a real Quantity, as $\frac{m}{a}$ is, which is abſurd. Now ſince abſolute Nothing multiplied into a real Quantity can produce no Effect. for the Reaſon and by the Argument now aſſigned, neither can it, when divided by a real Quantity, by the ſame Way of reaſoning, ſince Diviſion by a is the ſame with Multiplication by $\frac{1}{a}$. And ſince Multiplication and Diviſion are the ſame with Addition and Subſtraction reiterated as often as the Multiplicator and Diviſor implies: Therefore *abſolute Nothing* is ſuſceptible of none of thoſe *arithmetical* Applications that real Quantities are ſubjected to. q. e. d.

Corollary.

Abſolute Nothing therefore, when applied to real Quantities by Multiplication and Diviſion (and conſequently, when by Addition and Subſtraction) implies, that there is neither Product nor Quotient, that is, that the real Quantity is neither multiplied nor divided, but remains unaltered.

Propoſition XII.

In the *Analogy* of Things; as a *mathematical Point* is to *univerſal Space*, ſo is *abſolute Nothing* to the *abſolute and ſupreme Infinite*.

Demonſtrat. By *Prop.* 10. a *mathematical Point*, and *univerſal Space*, are the two *Limits* of material or natural Things; they involve ſimple, clear and diſtinct Idea's, and are as well known, as any of the Objects of human Knowledge. *Abſolute Nothing* is one of the Limits of the *univerſitas rerum omnium*, to wit, that in deſcending, below which nothing can fall; and therefore by the Rules laid down, for the *Analogy* of Things, the *abſolute* or *ſupreme Infinite* muſt be the other Limit, beyond which, nothing can riſe in aſcending, and between theſe two, all *Subſiſtences*, Finite, Indefinite, and relative Infinite are concluded. Wherefore by *Lemma* 3. and *Prop.* 9. as a *Mathematical Point*, &c. q. e. d.

Corollary I.

HENCE *abſolute Infinite* can neither be encreaſed nor diminiſhed. For *abſolute Infinite*

finite and *abſolute Nothing* being the *Limits* of all Things whatſoever, *abſolute Infinite* muſt be the greateſt of all Things, and ſo can neither be encreaſed nor diminiſhed, elſe it could neither be the greateſt, nor the aſcending *Limit*.

Corollary II.

HENCE *abſolute Infinite* is *One*, and *Individual*. Becauſe being neither capable of Encreaſe nor Diminution, and being the aſcending *Limit*, it muſt be one: And individual, becauſe there can be no other like it, it being the one aſcending *Limit*.

Corollary III.

HENCE *abſolute Infinitude* is only compatible to the divine Nature, and to nothing elſe. He being the greateſt of all Beings, *one*, and *individual*, or the ſingle poſſible Being of the ſame Nature.

Corollary IV.

Univerſal Space, is the Image and Repreſentation, in Nature, of the *divine Infinitude*. For ſince by *Prop.* 9. the created *Species* of Things are Images of the increated, and by *Prop.* 10. *univerſal Space* is the *greateſt* Limit of the viſible Creation, or material *Syſtem* of Things. Therefore *univerſal Space* is the natural Image of the greateſt Limit of all Things, or of the *univerſitas rerum omnium*; That is, by *Prop.* 12. and the *preceeding Corollary*, *univerſal Space* is the natural Image of the *divine Infinitude*.

Corollary V.

HENCE *univerſal Space* may be very aptly called the *Senſorium Divinitatis*, ſince it is the Place wherein all natural Things, or the whole *Syſtem* of material and compounded Beings, are preſented to the *divine Omniſcience*. Infinite Space is the Image of the *divine Infinitude*, wherein, as in a Picture of him (in whom all Things live, and move, and have their Being) all created Things preſent and manifeſt themſelves to the intuitive View of the *ſupreme Infinite*; and therefore in the *Analogy* of Things, infinite Space is to the *ſupreme Infinite*, what a human *Senſorium* may be ſuppoſed to be to Men.

Corollary VI.

AN *abſolute infinite Creature* is a Contradiction, becauſe *abſolute Infinite* is *one*, and *individual*, by *Corollary* 2. of this.

Scholium.

ONE very remarkable Difference between *Finite*, *relative* and *abſolute Infinite* is this. *Finite* may ſtill be increaſed by it ſelf, and yet continues its finite Nature, 'till the Number of Additions be actually Infinite. *Relative Infinite* may be increaſed, not by Finite, but by *it ſelf*, and ſtill continues its Nature, after *infinitely-infinite* Additions, without Bounds or Limits: For let it be ever ſo often increaſed by it ſelf, it continues but *relative Infinite*. But *abſolute infinite* can be increaſed by nothing, not even by it ſelf, elſe it could not be the abſolute or greateſt Infinite. In natural Things, *Finite*, and *relative Infinite*, never riſe higher than the third Power. The *Comets* and *Planets* move in *Orbits*, that are Curves of the *ſecond Power*. *Projectiles* move in *parabolick* Lines, which are of the ſame Order. The *Curvature* of the *Surface* of all the *celeſtial Bodies* are of the *conical ſection-kind*. The *Surfaces* of Fluids, riſing in great or ſmall Channels, are of the ſame Rank. Nature admits but of three Dimenſions, and I know of no natural Effect that riſes higher than the third Power. Even the *irrational Curves* that Nature forms, in conducting *ſolid Bodies*, and the *Rays of Light* (through different *refracting Mediums*) the ſhorteſt and eaſieſt Way: And in ſome other of her Operations, ſuch as the *Cycloid*, *Conchoid*, *Catenaria*, *Velaria*, *Curva Elaſticitatis*, *Logarithmica*, the *Spiral*, and the like, all of them are of the loweſt Rank of their Order: And their Natures imply only Portions of *Curves*, or of their *Area's*, of the *conical-ſection-kind*, the loweſt Order of all *Curves* whatſoever. So true it is, that Nature perpetually brings about her Purpoſes *the ſhorteſt and ſimpleſt Way*, and keeps conſtant to this beautiful *Analogy* of Things. But ſince Quantity may be ſtill increaſed, ∞ may be ſuppoſed to riſe to higher Powers than $\infty 3$ in the intellectual *Species* of Things, ſuch as are, $\infty 4$. $\infty 5$. ∞^{∞}. $\infty^{\infty^{\infty}}$ &c. Now tho' theſe Powers of ∞ ſuperior to $\infty 3$, can have no Place in natural Things, or the material *Syſtem*, yet are they the *intellectual Species* of created Things, and are in their reſpective Orders, *Images*, and Repreſentations of their *archetypal Idea's*, in the divine Mind, and Inſtances of the unexhauſtible Store of manifold Wiſdom in the divine Nature. And as *infinite Extenſion* is $\infty^{\infty^{\infty}}$ &c is the Image in Nature of *abſolute Infinite*, ſo $\infty^{\infty^{\infty}}$ &c. is the Image of the ſame abſolute Infinite, in the *intellectual Species* of Things. And by the *Analogy* of Things, relative Nothing, and relative Infinite are the *intellectual Images* of *abſolute Nothing*, and *abſolute In-*

finite, ſo univerſally does this *Analogy* hold good.

Propoſition XIII.

Abſolute Infinite, in the *Analogy* of Things, is the preciſe and proper Oppoſite to *abſolute Nothing*.

Demonſtrat. This is plain from *Prop.* 12. and its *firſt Corollary*. For ſince a *mathematical Point* is the preciſe and proper Oppoſite to univerſal Space, and ſince in the *Analogy* of Things, a *mathematical Point* is to *infinite Space*, as *abſolute Nothing* is to *abſolute Infinite* : Therefore *abſolute Infinite* is the preciſe and proper Oppoſite to *abſolute Nothing*. Beſides, *abſolute Nothing* and *abſolute Infinite*, being the *Limits* of the *Univerſitas rerum omnium*, in aſcending or deſcending from created finite Beings, they muſt be preciſe and proper Oppoſites, having nothing common but their being *Entia and Limits*. q. e. d.

Corollary I.

SINCE *abſolute Nothing*, in its poſitive *Idea*, implies the utmoſt Impoſſibility of, and the moſt extreme Contradiction to *Reality* or real Being, as it moſt certainly does; (for it is not poſſible to conceive a greater Contradiction to, or Impoſſibility of *Reality* or real Being, than is implied in the poſitive Idea of *abſolute Nothing* or *Non entity*, beyond which there is no further deſcending from Reality or Being; it having no real *Entity*, though it may be claſſed, as to its negative Conception, in the *Category* of *Entia rationis*:) Therefore by the *Analogy of Things*, *abſolute Infinite* muſt imply in its poſitive *Idea*, and that neceſſarily, the utmoſt Poſſibility, and the moſt extreme Neceſſity of *Reality* or real Being.

Corollary II.

WHEREFORE ſince *abſolute Nothing*, in its poſitive *Idea*, implies the utmoſt Contradiction to real Being, ſince *abſolute Infinite* is the preciſe Oppoſite to *abſolute Nothing*; ſince alſo being Oppoſite in every Thing (except as *Entia* and *Limits*, which do not affect the Realities in *abſolute Infinitude*, as to its poſitive *Idea*) *abſolute Infinitude* muſt imply in its poſitive *Idea* the utmoſt neceſſity of *Reality*, or real Being; therefore of Neceſſity the *abſolute Infinite* muſt really *Be*, or the Being to which *abſolute Infinitude* is only compatible, muſt neceſſarily exiſt.

Propoſition XIV.

Finitude and *Infinitude*, when applied to natural and created Things, in their poſitive *Idea's*, imply not *Realities*, but the *Modes* of *Realities*

Demonſtrat. *Finitude*, when applied to natural or created Things, imports only in it's poſitive *Idea* the Proportions of the ſeveral Degrees of *Affections*, or *Properties*, of theſe Things to one another. *Infinitude*, the Unboundedneſs of theſe Degrees of *Affections*, or *Properties*. *Finitude* and *Infinitude*, in themſelves, abſtracted from a proper Subject or *Suſtantive*, are incomplete *Idea's* in natural or created Things. *Infinite Extenſion*, *Number*, *Duration*, *Wiſdom*, *Knowledge*, &c. are compleat *Idea's*, whereof theſe *Realities* are the Subjects or *Subſtantives*, and the *Infinitude* the *Epithet* or *Adjective*: Wherefore *Finitude* and *Infinitude* in natural or created Things, being but *Adjuncts* to *Realities*, in their poſitive *Idea's*, do not imply *Realities*, but the *Modes* of *Realities*. q. e. d.

Corollary I.

HENCE the *Modes* of natural or created Things are *Realities*, in the ſupreme or *abſoſolute Infinite*, or the *Modes* of natural or created Things, when *analogically* elevated to the ſupreme and *abſolute Infinite*, are in him not *Modes*, but the utmoſt *Realities*. This is plain from the *preceeding Propoſition*, and the *2d Corollary* of the 13. *Infinitude* is but a *Mode* in natural or created Things, but in the ſupreme or *abſolute Infinite*, it implies in its poſitive *Idea* the utmoſt *Reality*, to wit, neceſſary Exiſtence; and the ſame muſt of Neceſſity be true, in all thoſe Attributes of the divine Nature, which *analogically* derived down to the Creatures, are in them but *Modes*, in him they are the utmoſt *Realities*; becauſe they are all complicated and affected with, or (ſo to ſpeak) multiplied into *abſolute Infinitude*, which *realizes* thoſe *Modes* of Creatures, and *tranſubſtantiates* them into poſitive and *real* Qualities. So true is the *metaphyſical Axiom*, *quicquid in Deo, ipſe Deus eſt*. I do not here contend that there may not be *acceſſory Idea's* in the divine Intellect, or that there may not *arbitrarily* ariſe in the divine Intellect *Images* of Beings, whoſe Exiſtence in Fact, is not neceſſary. Since the *Idea* of a Creature cannot be neceſſary to him, or elſe they would neceſſarily be. But even thoſe *acceſſory* or *arbitrary Idea's*, in the divine Imagination, by being there, become of a quite different Nature from the like in Creatures, for by being there, they (by Virtue of his *abſolute Infinitude*) receive a Being infinitely ſuperior to the like *Ideas* in created Beings, not neceſſarily, but with infinite Freedom and Liberty. And thoſe Affections and Properties in Creatures,

tures, which in them are but *Modes*, when *analogically* carried up to the like or ſimilar Affections or Attributes in the divine Nature, are in him the utmoſt *Realities*, as being complicated with *abſolute Infinitude*, and thereby transformed or exalted into real Quantities or actual Suſiſtences.

Corollary II.

HENCE *Power*, *Subſiſtence*, *Duration*, *Knowledge*, *Wiſdom*, *Goodneſs*, *Beauty*, &c. which in created intelligent Beings are the Images of *Omnipotence*, *neceſſary Exiſtence*, *Eternity*, *Omniſcience*, *the divine Sophia*, *Benignity*, *infinite Perfection*, &c. in the divine Nature, and are but *Modes* of Being, and not eſſential Affections in theſe; are in him infinite *Realities*, and living active Principles: And he that would reaſon *analogically*, from the Nature of theſe in created intelligent Beings, to the Nature of thoſe in the divine Eſſence, without having the utmoſt Regard to the *abſolute Infinitude*, which in a Manner quite changes the Nature of the former, and exalts them into a different *Category*, would be miſerably miſtaken. For Inſtance, he that, becauſe the Duration of natural Things is ſucceſſive, would conclude ſo of the divine *Eternity*; He that would reaſon, becauſe human Power cannot give Being and Subſtance to that which had none, the divine *Omnipotence*

tence could not: He that would infer, that becauſe the Knowledge of rational created Beings is *progreſſive*, the divine *Omniſcience* were alſo progreſſive, muſt be egregiouſly miſtaken: Becauſe in this *Analogy* he does not take in the *abſolute Infinitude* which elevates and exalts the Duration, Power and Knowledge of created Things, into a Degree of Reality of which theſe are but the Images or Pictures. He that from the Picture of a Man would *analogically* reaſon about human Nature, and from the blending and Poſition of ſome Colours on Canvaſs, would reaſon to Life, and Knowledge: Or from the reflected Image of the Sun in Water, would conclude of Light, and Heat, could not err more groſsly.

Scholium.

IN intelligent compounded Beings, the *Powers* belonging to the Body are not only finite, but very low in the Order of Finites. The *Eye* perceives not diſtinctly a very large nor ſmall Object. Too ſtrong a Noiſe ſtunns the *Ear*, and one too weak does not act upon the *Organ*: Neither of them produces a diſtinct Hearing; and it is ſo in all the other Senſes. Thoſe *Organs* are ſo contrived, as to perceive beſt the ordinary Effects of common Life, the Objects that Neceſſity of Subſiſtence do moſt readily preſent to us. Theſe they are fitted for, and little elſe; they have a wonderful Facility in manifeſting theſe diſtinctly to our Minds, and have a juſt Proportion to the Objects about us; but ſeem not contrived nor deſigned for Curioſity, or conveying much more Knowledge to us, than what the Conveniencies of Life require: Elſe infinite Wiſdom and Power might have eaſiſily formed them ſo, as not to need thoſe Helps and Aſſiſtances, which we are obliged to employ, when we aim at any more particular Knowledge of the intimate Natures of the Things about us, than Conveniency makes neceſſary or commodious. The *Faculties* belonging to the rational Soul are likewiſe finite, but of a higher Rank of Finites than thoſe Powers belonging to the Body. The *Imagination* can paint a larger *Idea*, than the *Eyes* can ſee, and the *Memory* lodge a greater Store of Images, than all the Senſes can preſent at one Time: And the Underſtanding can combine and disjoin theſe, and compare them many different Ways: But ſtill theſe Faculties are but finite in their *Capacity*. We can form no diſtinct Idea's of *Millions* of *Millions*, of a *multangular Figure*, or any *relative Infinite*, ſmall or great: The Imagination ſeems not able to contain theſe; there is no room on it, for ſo large Pictures. The *Memory* is the *Repoſitory* of the Images that have been framed on *Phantaſy*, and can go no further than it reaches; nay ſeldom or never contains more than a ſmall Part of thoſe. The *Underſtanding* can work no further than theſe two afford Materials; its Works being to *collate*, *combine*, and *garble*, as it were, theſe Images and *Idea's* the *Imagination* and *Memory* preſent to it. All theſe are limited, as the Senſes are, though not quite ſo ſtraitly, because the Underſtanding may variouſly combine thoſe *Idea's*, they have conveyed to the *Imagination* and *Memory*, and ſo increaſe their Number: And the *Will* having no ſubject Matter to proceed upon, but as it is prepared by theſe prior Faculties, can go no further than thoſe allow it. The Underſtanding may diverſify theſe as far as their *Combinations* will reach; and the *Will* may pick and chooſe among theſe; but ſince it cannot *create* Objects for it ſelf to work on, it muſt be limited to the Images and *Idea's* on the *Imagination* and in the *Memory*. The *Will* I mean, as it is a Faculty belonging to the *rational Soul*. All theſe Faculties ſeem to have been originally deſigned for Nothing but this *material World*, and the Syſtem of Things about us. They help us to no Notion or Conception of any Sort of Beings diſtinct from *Matter*, but in ſo far as *Analogy* will bear us out; and even as to the *material World*, they ſeem fitted for little elſe, beſides the groſſer, more general and more neceſſary Knowledge of Things that are required for due Conveniency, or Subſiſtence, and ſhew but ſome of the groſſer *Out lines* of the real Natures of Things. Every Thing that might violently entertain our Curioſity, or flatter our Vanity, as to the Knowledge of the Nature of Things, ſeems to have been induſtriouſly concealed from us, and no Faculties to have been indulged us for theſe Purpoſes, leaſt they ſhould have withdrawn us too ſtrongly from Things of greater Moment to the End of our Being: Elſe infinite Wiſdom and Power had contrived them after a more perfect Manner, with a larger *Capacity*, and a ſtronger *Energy*. As to the Faculties of the *ſupreme Spirit*, (which is a third Part of intelligent compounded Beings) they moſt certainly are infinite in their *Capacity* and *Energy*, I mean they may be enlarged and increaſed without Bounds or Limits, which by *Definition* 4. is to be relatively infinite. Not only the *Acts* of theſe Faculties may be multiplied perpetually, but the *Capacity* and *Energy* of theſe may be dilated and intended without Bounds or Limits. The *Perception*, the *Deſires*, the *Will*, (the Faculties belonging to the *ſupreme Spirit*) are unlimited and boundleſs, fitted and deſigned for *infinite Objects*. Theſe indeed are the firſt, *principal*, and *original*

ginal Faculties, belonging to all compounded intelligent Beings: By which they are made capable of communicating with the *ſupreme Infinite*. And next in order of Nature to the *ſupreme Spirit*, is the *rational Soul*, whereby they are enabled to communicate with the *material World:* And to the Faculties of this *ſecondary* Part of the Compoſition, the Senſes of the Body are *Conduits*, and Conveyances, which make up the third and laſt Part of compounded intelligent Beings: In the due *Subordination*, the perfect *Harmony*, and perpetual *Concord* of theſe Three, with each other, the *Perfection* of theſe Beings does conſiſt: In their *Diſcord*, *Confuſion*, and *Rebellion* one againſt the other, their *Degeneracy*, *Corruption*, and *Fall*. It can be no Difficulty to thoſe, who are acquainted with the *Analogy* of Things, to conceive how theſe ſeveral Parts of compounded intelligent Beings are contained, without Confuſion or Contrariety, in each other, in their primitive and uncorrupted State; when they conſider, that in Water is contained Air, in that *Æther*, in that Light, and perhaps in this laſt, a more ſubtle and refined *Spirit*; and all theſe in perfect *Harmony* and *Concord*. But to conſider theſe a little more particularly. The *Perception*, as it belongs to the *ſupreme Spirit*, muſt of Neceſſity be a *paſſive* Principle, becauſe it cannot create its Objects, but receive thoſe preſented to it: That it is infinite, is plain, becauſe its adequate Object is infinite, and the *ſupreme Infinite*; it being beſtowed on the intelligent Beings, in order to communicate with the *abſolute Infinite*. The Neceſſity of this third Part of the Compoſition of the mentioned Rank of intelligent Beings ſhall be afterwards demonſtrated; I proceed to conſider in a few Words the Nature of ſome other of theſe Faculties, belonging to the *ſupreme Spirit*.

Propoſition XV.

THE *Deſire* is infinite in its *Capacity*, the moſt *cardinal*, moſt *quick*, and *ſenſible*, and moſt *active* Faculty of the *Mind* or ſpiritual Part of compounded intelligent Beings, and the *Will*, and the *Affections* are but *Modifications* of it.

Demonſtrat. To be convinced of the Truth of this *Propoſition*, we need only reflect on the Source of all the Happineſs or Miſery of intelligent Beings, and we ſhall find it ariſes from the Enjoyment or Diſappointment of their *Deſires*. There is in all intelligent Beings a reſtleſs *Appetite* or *Deſire* of Happineſs: From the Moment of their Being, through all the Ages of Eternity, all their *Labour*, and *Travel* is for this Purpoſe: Nor are they devoid of it, either immediately in the *End*, or mediately in the *Means*, for one Inſtant of Time, in all their endleſs *Duration*. Now this is the neceſſary Effect of the Faculty of *Deſire*, no Object leſs than *Infinite* can ſatisfy it. For let it be ſuppoſed to have come to the Poſſeſſion of any Object leſs than *Infinite*, its plain, it can *deſire* yet a greater, and a greater, without Bounds or Limits; that is, it can deſire an *infinitely great Object*, that is, the *Deſire* it ſelf is infinite in its *Capacity*; its *Acts* are *inſtantaneous*, and its Enjoyments or Diſappointments for a Time ſwallow up the *Acts* of all the other Faculties: And therefore it is the moſt *quick* and *ſenſible*: It ſets all the Powers of the whole Compoſition on Action, to obtain its *Ends*, and therefore it is the moſt *active*: And upon all theſe Accounts, the moſt *cardinal* Faculty of the *Mind*: Chooſing or refuſing, that is *willing*, is but the Deſire applied to a particular Object; the *Affections* are but the *Complexions* of the *Deſire*, as applied to this particular Object: And therefore are both but *Modifications* of the *Deſires*, wherefore, *&c.* q. e. d.

Scholium.

To apprehend how infinitely capacious, active, and ſenſible the Deſire is, we need only imagine our ſelves, ſeparated from the Objects of Senſe, and the preſent *Amuſements* of *Life,* with all the Faculties of the Soul *awake:* And we ſhall then be able to conjecture how *ſtrong, active, reſtleſs,* and *unſatiable* our Deſires would be. So as to ſwallow up and extinguiſh all the other *Acts* of the Faculties of the whole Compound. Thoſe only can moſt ſenſibly feel the Force of this Reaſoning, who have in ſome Meaſure, and for ſome Time been in this State.

Corollary I.

SINCE the Deſire is *infinite* in its Capacity, and may be dilated beyond any finite Object, how great ſoever, it is evident it cannot be over-filled, or ſuper-abundantly (ſo to ſpeak) ſatisfied with any Object leſs than *infinite:* Since alſo, the greateſt relative Infinite, cannot be aſſignable by *Corollary* 1. *Prop.* 1. Therefore the *Deſire* cannot be adequately and over-filled with any leſs Object than the *abſolute Infinite;* it being capable of being enlarged beyond the Dimenſions of any *relative* or *creaturely Infinite* aſſignable: And ſince by *Corollary* 6. *Prop,* 12. no *Creature* can be *abſolutely Infinite,* therefore the Deſire can be perfectly and adequately filled and ſuper-abundantly ſatisfied, by nothing leſs than the *ſupreme* and *increated Infinite.*

Corollary II.

SINCE the *Deſire* is *infinite*, when filled and ſatisfied with its proper and *adequate Objects*, it muſt be infinitely happy: For ſince Nothing, by the *preceeding Corollary*, but the *abſolute* and *increated Infinite*, can adequately fill, and ſuper-abundantly ſatisfy it; the *abſolute* and *increated Infinite* muſt be its proper Object, and the *Deſire infinite* in it ſelf, filled and perfectly ſatisfied with its proper Object, the *ſupreme* and *increated Infinite*, muſt of Neceſſity be infinitely happy. Happineſs ariſing from the *Congruity* of the Object with the Faculty, and this *ſupreme* and *increated Infinite* being the proper, and indeed the only proper Object, (ſince the ſupreme Infinite is one) it muſt alone be the *congruous* Object, and ſo the Faculty being *infinite*, the Object *infinite*, and they infinitely *congruous* to one another, the *Deſire*, in the Poſſeſſion or Enjoyment of this Object, muſt of Neceſſity be infinitely happy.

Propoſition XVI.

IN regard of *intelligent Beings*, every Thing that is in the *Univerſitas rerum*, may be fully comprehended under theſe three general Heads, to wit, the *Faculty* or *Deſire*, the Object of this *Faculty* or *Deſire*, and the *Senſation* ariſing from the *Congruity* or *Incongruity* between theſe two.

Demonſtrat. All the diſtinct Subſiſtences that actually exiſt, are comprehended under theſe two general Heads, the *ſupreme* and *abſolute Infinite*, and the *Creature*, and theſe can be only Objects of the *Faculty* or *Deſire*. The *Faculty* or *Deſire* being infinite, by the *preceeding Propoſition*, may contain or receive both theſe, and the *Senſation* ariſing from their *Congruity* or *Incongruity* to the Faculty, muſt together with theſe two general Heads mentioned, comprehend every Thing in the *Univerſitas rerum* in regard to any particular intelligent Being. For nothing can be imagined in the whole Extent of Being, *real* or *intellectual*, that may not be reduced to *Faculty*, *Object*, or *Senſation* ariſing from theſe. Therefore, *&c.* e. q. d.

Propoſition XVII.

AN *intelligent Being*, compounded of a Body, Soul, and Spirit, with proper Relations, and in Subordination to each other, is a real *Epitome*, *Image*, or *Repreſentation* of the *Univerſitas rerum omnium*.

Demonſtrat. This is ſo evident from the *Analogy* of Things, that there can be no

Difficulty in it, to thoſe who will conſider, that the *ſupreme Infinite* could have nothing more perfect than himſelf and his other Works, and his own perfect Adminiſtration and Government of theſe, to form this compounded Being upon. He muſt be conſiſtent with himſelf; and his *compounded Works*, in their component Parts muſt reſemble in the *leſſer* Compoſitions, the *ſimilar* Parts of the *greater*. This *intelligent compounded Being*, being to have a *material* Part, what can it have more worthy of the Work of infinite Perfection, than his *greater Syſtem* of material Beings to reſemble? His Soul or rational Part can reſemble nothing more worthy of *Him*, than the other higher Orders of created Spirits: And his *ſupreme Spirit* or Mind, will bear its beſt Reſemblance, when it repreſents the *ſupreme Infinite*. His Adminiſtration and Government of the whole *Syſtem* of created Beings, can be repreſented by nothing ſo aptly, as the Relations and Subordination of theſe to each other, and to the reſt of intelligent Beings, which theſe could only be formed upon. In a Word, it is impoſſible that any *Idea* of a compounded intelligent Being, made up of ſeveral Parts, each *ſimilar* to ſome greater Part of the *Univerſitas rerum*, already exiſtent, could enter to the divine Mind, which could be more perfect and more worthy of of *Him*, than this, that each inferior Part ſhould be an *Epitome* and Reſemblance of his Works already made, and the higheſt Part, of *Himſelf*; and that their Relations and Subordination ſhould reſemble his Adminiſtration of the Whole: I ſay it is impoſſible it ſhould be otherwiſe, to preſerve his Conſiſtency with himſelf, and to carry on this *Analogy*, through every individual Part of his Works, through the whole *Scale* of Beings, as it moſt certainly is carried. And this is not a meer *metaphorical* Picture, and Reſemblance, but the real and phyſical Nature of compounded intelligent Beings. Wherefore, *&c.* q. e. d.

Corollary I.

From this Foundation, by a proper *Analogy*, with the due Limitations, all the *Relations* and *moral Duties* of *intelligent compounded Beings*, to the *ſupreme Infinite*, to *ſimilar intelligent Beings*, and to *themſelves*, may be eaſily deduced; for Inſtance, do we reſemble the *ſupreme Infinite*, in our *ſupreme Spirit*: Then as he is the firſt in *Order* of *Beings*, ſo is this Part of our Compoſition the firſt and moſt principal, in our *Order* of Parts. All the reſt muſt be ſubſervient and ſubordinate in us, to this, as the reſt of the ſeveral Ranks of Beings are to him: This we are chiefly and mainly to cultivate by fitting it up for him, for whom it was *originally* deſigned and beſtowed upon us. Here, we muſt aſpire after *Him*, and open our *Deſires* for *Him*, by a *Love* worthy of *Him*, ſuperior infinitely to all our other *Loves* and *Deſires:* On this Part and its Faculties, our greateſt, our chief, nay, and our only Labour is to be beſtowed; that it may be expanded, dilated, and elevated to its proper Rank in the *Order* of our Parts; that the due Subordination may be reſtored; which done, all the other Parts will perform their proper *Functions*, in *Harmony* and *Concord*. In regard to other *intelligent Beings*, we are to conſider them as like *Images* of *Him* and his *Works*; and to follow them with a *Benevolence* proper to ſuch *Images*, to imitate his Conduct of *Love* and *Forbearance*, to all his Creatures. But this only by the By.

Corollary. II.

HENCE, in a proper *Analogy*, the Nature of the *ſpiritual* and *material* Parts of compounded intelligent Beings and of the *Union* between theſe, as far as they may be known by mere human Reaſon, is to be deduced. Compounded intelligent Beings are *Epitomes* or *Images* of the *Univerſitas rerum*. In their Bodies they reſemble the *material Syſtem* of Things, in their *ſpiritual Parts* they reſemble the ſpiritual World, the Union of theſe two is a Reſemblance of (or is maintained and preſerved after) the Manner, the *ſupreme Being* governs the *material Syſtem* of Things: Who being intimately preſent with every individual *Atome* of Matter, yet more *eminently* acts from his ſuperior *Throne of Glory*, having the whole *Syſtem* of Creatures in one View preſented to him, in the *univerſal Space*, his ſpecial *Senſorium*. By this Principle, as a *Key*, the whole *Philoſophy* of human Nature, of the animal, rational, and divine Life, of the Paſſions and Affections of the Soul, and even of the *Organiſm* of the Body, ſo far as it is juſt and genuine, and given to mere human Reaſon to know, is to be unlockt, and that not *metaphyſically*, but *phyſically* and in reality. But who is ſufficient for the Detail of theſe Things?

Corollary III.

HENCE, the *Immortality* of the ſpiritual Part, of intelligent compounded Beings, is evidently to be deduced. For ſince the *ſpiritual* Part of intelligent compounded Beings, is an *Epitome* and *Image* of the *ſpiritual World*; and the *ſupreme Spirit*, (that

Part which God *originally breathed* into Man) is an *Epitome* and *Repreſentation* of the *abſolute Infinite*; ſince by *Corollary* 2. *Prop*. 13. he neceſſarily exiſts, therefore the Soul, or *ſpiritual* Part (whereof this ſupreme Spirit is the *Fund* or *intimate Subſtance*) muſt exiſt for *ever*. Not neceſſarily, for that Concluſion would drop the proper Limitation in the *Analogy* ariſing from *abſolute Infinitude*: But, as being *Images* of him, who exiſts *neceſſarily*, and having their Being from him. Their *Immortality* is indeed an active, *living Principle*, not of neceſſary, (but *derived*) Exiſtence. They are *immortal*, as having repreſented on them all his communicable Perfections; of which, *perpetual* Exiſtence is one, though *neceſſary* Exiſtence be not; that being contrary to *Creation*, or deriving a Being from another. *Immortality* in Creatures is nothing but *perpetuity* of Exiſtence, and if Exiſtence at all be communicable to Creatures, *perpetual* Exiſtence muſt be communicable: For Exiſtence, being an *active*, *living Principle*, will of it ſelf continue *Being* for ever, unleſs it be deſtroyed: Which is impoſſible, both from the *Immutability of God*, and the Nature of his own *Immortality*, of which this is an *Image*, or *Reſemblance*. The *Immortality* of ſpiritual Creatures, is an *Emanation* from an *Image* of the divine communicable *Immortality:* And muſt reſemble every Thing in it, but *Neceſſity*, that being a Contradiction to its being *derived*. But in every other Circumſtance it perfectly reſembles the *Immortality* of the ſuperior Infinite, (as far as creaturely Properties can reſemble divine Attributes) and ſo cannot poſſibly be deſtroyed: And this is the true Source of the *Immortality* of all Cteatures. It is true, this *Demonſtration* equally concludes the *Immortality* of all his other Works, ſince they are equally his *Images*, in a higher or lower Degree of Perfection: And without all doubt it muſt be ſo; *for the Gifts and Calling of God are without Repentance.* And this is the genuine, and as the *Schools* call it, the *apodeictick Demonſtration* of the *Immortality* of all the Works of God, under ſome Form or another; it is from their being his *Images* that their Immortality ſprings.

Corollary IV.

In the *Analogy of Things*, the *Deſire* being the *cardinal* Faculty of the Soul, and *infinite* in its *Capacity*, is as the *infinite Space*, to the *divine Plenitude:* Which infinite Space, nothing created can adequately fill, but the divine *Plenitude.* And in this View, the *infinite Capacity* of the *Deſire* may be conſidered as a *boundleſs Void*, made to receive ſome *fleeting*, limited Parts or *Syſtems* of Matter, acting by eſtabliſhed Laws, and in comely Order, upon one another for a while; but can never be perfectly filled, or adequately repleniſhed, but by the *ſupreme Infinite:* Who is preſent with, and repleniſhes every Point of the great and *univerſal Void* of Nature.

Scholium. I.

As in regard to intelligent Beings, the *Univerſitas rerum omnium*, is fully comprehended under theſe three general Heads; the Faculty or *Deſire*, the *Object*, and the *Senſaſation* ariſing from the *Congruity* or *Incongruity* between theſe: So the ſame *Analogy* with proper Limitations is preſerved in the material *Syſtem* of Things. For anſwerable to theſe, we have in the *material World Gravitation*, which wonderfully *analogiſes* to the *Faculty* or *Deſire* in the *ſpiritual World*, and this to that; both being the *active*, *cardinal*, and *energetick Principles* of either *Syſtems* reſpectively. Next we have a *Maſs* of extended ſenſible Matter, if collectively conſidered, or *Syſtems* of *material Bodies*, diverſely figured and ſituated in regard to one another, if ſeparately conſidered: And in both theſe Views, they admirably repreſent the Subject or Object of the

Deſire, which is analogiſed by *Attraction* or *Gravitation.* And laſtly, we have the *harmonious, comely,* and *decent* Motions, and Actions of one Body upon another, ariſing from the *Attraction* or *Gravitation*'s being impreſſed on the *Maſs*, and each particular Body; and this too anſwers wonderfully to the *third* Principle in the *Univerſitas rerum.* And this *Analogy* deſcends even to particular Bodies, for in theſe we have the Form, the ſubject Matter, and the *Congruity* between theſe.

Scholium. II.

FROM the whole preceeding Chain of *intellectual Truths,* we may form to our ſelves ſome *faint, low,* and *imperfect Image* or Repreſentation of the EVER-BLESSED TRINITY IN UNITY. For ſince by *Corollary* 1. *Prop.* 14. the *Modes* of natural or created Things, when *analogically* elevated to their ſimilar Attributes in the *ſupreme infinite,* in *Him,* are infinite *Realities:* Since, by the *ſame Propoſition,* an intelligent Being is, as to its ſpiritual Part, an *Emanation* from, an *Image* and *Repreſentation* of, the *ſupreme Infinite:* And ſince alſo, in regard to theſe intelligent compounded Beings, all that is in the *Univerſitas rerum omnium,* may be fully comprehended under theſe three general

Heads or Principles, the *Deſire*, the *Object*, and the *Senſation* ariſing from the *Congruity* or *Incongruity* between the *Faculty* and the *Object*: Since laſtly, this *Analogy* is preſerved full and clear, through the *ſpiritual* and *material* Worlds, and each particular Body, that is, through the whole *Syſtem* of Creatures: It is highly probable, ſo conſtant and univerſal an *Analogy* can ariſe from nothing, but from its *Pattern* and *Archetype* in the divine Nature: And without all peradventure, could we fully and clearly carry it up to its *Source*, we ſhould there find the *Origin* and *Source* of this HOLY TERNARY; or of theſe three eſſential Relations, of the Whole, to the Whole in the completion of the GOD-HEAD. Let us therefore try, in ſome poor low Manner, to carry up this *Analogy*, as high as poſſible we can. The firſt Thing then that we are to conſider in the divine Nature is, the *Deſire*; now this being ſuppoſed, to belong to a *ſupremely infinite intelligent Being*, muſt be an infinitely *active*, *ardent*, *ſtrong*, and *powerful Thought*: And that, not as created, or *relative Infinitude* expreſſes it, but as the abſolute and *ſupreme Infinitude* elevates this *Deſire*. Now then this ſupremely infinite *Deſire*, this *active* and *ardent Thirſt* after Happineſs, or after a full, plenary, and compleat, beautifying Object, we ſhall ſuppoſe to repreſent the FATHER, the *original* and firſt *Principle* in the divine Nature: Seeing then, there is no Conſideration here of any Thing but of *God himſelf*, nor is it poſſible for any other Object, but *God himſelf* to ſatisfy, and adequately to fill this *ſupremely infinite Ardor*, *Thirſt*, and *Deſire* of *Happineſs*. Therefore *He himſelf*, reflected in upon *Himſelf*, viewing and contemplating his *own* infinite Perfections: The *infinite ardent Deſire*, filled and ſatisfied with his *own* eſſential Happineſs: The *Brightneſs of his Glory*, and the *expreſs Image*, (the eſſential *Idea*) *of his Subſtance*, reflected in upon *himſelf*: Or *God himſelf reduplicatively* contemplating *himſelf*, (he having nothing elſe poſſible, to be the Object of his *Love*, *Delight*, or *Deſire*) repreſents to us the *begotten Deity*, the SON, the ſecond divine *Principle* in the Order of the *Godhead*. Upon this *Reflection*, *Contemplation*, and Poſſeſſion of *God himſelf*, in *himſelf*; there muſt of Neceſſity ariſe *a Joy*, *Happineſs*, *Acquieſcence*, and *Satisfaction* of *God himſelf* within *himſelf*, ſo much the more *perfect*, *full*, *extreme*, and *infinite*, as his *Deſire*, *Thirſt*, and *Ardor* after Happineſs was *active* and *ſtrong*: And this may ſhadow out to us the third and laſt in Order of theſe eſſential *Principles* in this *myſterious Ternary*, to wit, the HOLY GHOST.

And though theſe three Relations of the *Godhead* in it ſelf, in regard to Creatures, and when derived down *analogically* to natural Things, may appear but *Modifications* of a real Subſiſtence; yet in regard to the *divine Nature*, and conſidering his *ſupreme Infinitude*, they muſt be *eſſential* and infinitely *real* and *living Principles*; and in this *Image*, and View of the HOLY AND UNDIVIDED TRINITY, *low* and *poor* as it is; it is impoſſible the SON ſhould be without the FATHER, or the FATHER without the SON, or both without the HOLY GHOST. It is impoſſible, the SON ſhould not *neceſſarily* and *eternally* be begotten of the FATHER, or that the HOLY GHOST ſhould not *neceſſarily* and *eternally* proceed from both; *He* neceſſarily ariſing from the *Senſation* of the infinite Agreement and *Congruity* of the *Object* with the *Deſire*. And though the *Idea*, *Image*, or *Repreſentation*, that *God* makes of *himſelf* to *himſelf*, be the ſame *God* in *Eſſence*, ſince it is a moſt perfect, expreſs, and ſubſtantial *Image*, or *Repreſentation* of the whole divine Subſtance and Nature, and *neceſſarily* has in it the Whole, that is in *God*, and with the ſame infinite Perfections, that are really in the *Godhead*, being only the *Divinity*, reflected in upon it *ſelf:* Yet there is a Difference, between the *Idea*, *Image*, and *Re-*

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preſentation, and the *original Divinity*. For the ſame Perfections which are in the *original* and *contemplating Divinity directly*, are but *indirectly*, and by Reflection in the *contemplated* and *begotten Deity*; and they differ by the *Relations* of *begetting*, *contemplating*, and *repreſenting*; and being *begotten*, *contemplated*, and *repreſented*: And though theſe be only *mere Relations*, and *Modifications*, when transferred to natural and created Things, yet are they *infinite Realities* in the *Deity*. It is the ſame with the *Senſation* of *Love*, *Joy*, *Acquieſcence*, and *Happineſs*, that ariſes in the *divine Nature*, from *contemplating* and *poſſeſſing himſelf* and his own infinite Perfections within *himſelf*. And theſe *three Relations* of *contemplating*, being *contemplated*, and of *Acquieſcence* ariſing from them; which in a natural View, would only be *Modifications*; yet in the *divine Nature*, are infinite *Realities* and *eſſential* and *living Principles*: And may ſerve as a faint and imperfect *Image* of THIS ADORABLE AND UNSEARCHABLE MYSTERY. Now though theſe Views and Repreſentations of this INEFFABLE AND INCOMPREHENSIBLE MYSTERY ariſe *naturally* and *neceſſarily* from the *Analogy* of Things, ſufficiently eſtabliſhed in the *preceeding Propoſitions*; and without all doubt, have ſome *real Truth* in them; ſince it is certain, theſe *Re-*

Relations and *Modifications* of Subſiſtence are really in *intelligent Beings*, in the *material Syſtem* of Things, and in *particular Bodies*; and could have no other *Riſe* and *Source*, but from their *Patern* and *Archetype* in the *divine Nature:* Yet when applied to the *ſupreme Infinite*, theſe Relations and Diſtinctions muſt be expreſſed in Words, that have a quite different Senſe affixed to them, by common Uſe, from what they ſhould here *import:* And ſince the *Attributes* and *Relations* of the *ſupreme Infinite*, muſt be *incomprehenſible* to *finite Creatures*, eſpecially to *Creatures* converſant only about low, groſs, and material *Images*; all I would be underſtood to conclude from this *Analogy*, thus raiſed to the *ſupreme Infinite*, is, that *Reaſon* may form an *analogical* imperfect *Image* (and that is all it can do) of this INCONCEIVABLE MYSTERY, which may in ſome Meaſure help thoſe (who have not attained to a more perfect Guide or higher Lights) to believe the poſitive Relations of his own Nature, by *God himſelf*, though they be not able perfectly to comprehend or expreſs them. And if this poor Repreſentation of ſo PROFOUND A MYSTERY, ſo *certainly* and FULLY revealed in *holy Writ*, can, by the *divine Bleſſing*, any wiſe contribute to *ſettle* and *quiet* ſober and honeſt Minds, I ſhall have my full

Intention; Nothing leſs than *Omnipotence*, and *Omniſcience*, being ſufficient to deal with thoſe who are otherwiſe diſpoſed.

Propoſition XVIII.

THERE muſt of Neceſſity be ſome *Principle* of *Action* in intelligent Beings, *analogous* to that of *Attraction* in the material *Syſtem*; and that is, the *Principle of Reunion* with the *ſupreme Infinite*, by him originally impreſſed on their *ſupreme Spirits*.

Demonſtrat. That there muſt be ſome Principle of *Action* impreſſed on intelligent Beings, *analogous* to that of *Attractiou* in the material *Syſtem*, is evident from the *Analogy of Things*, the Conſiſtency of the Works of the *ſupreme Infinite* with themſelves, the *Uniformity* conſpicuous in all the Creatures, and that the Material are but *Images* and *Repreſentations* according to their reſpective Natures of the *ſupreme Infinite*, as well as of the intelligent *Syſtem* of Beings. There muſt therefore ſome *great Principle* of Uniformity run through both *Syſtems*, that is, the whole Creation. Now, that *Attraction*, or ſomething *analogous* thereto, is the great Principle of *Activity* in the material *Syſtem*, has been ſufficiently demonſtrated in the former Part of this Treatiſe. There muſt of Neceſſity therefore be ſome *great Prin-*

Principle analogous to to this, in the *Syſtem* of intelligent Beings. And that this can be nothing, but that great *Principle of Re-union* with the Author of their Being, originally impreſſed on every intelligent Creature, is evident from hence, to wit, that the *ſupreme infinite Being*, infinitely powerful and perfect, muſt neceſſarily ſubject, draw, and unite all intelligent Beings to himſelf, to make them as happy, as their reſpective Natures can admit. That himſelf is the *ſole Object* of their Happineſs cannot be doubted; there may be collateral and acceſſory Objects of their Happineſs, but even theſe muſt flow from him: But that he is the ſupreme Object, and the ſingle one that can adequately ſatisfy them, has been ſhewn before: That therefore, to bring them to this Happineſs, he muſt impreſs upon, or derive to them a *Principle of Re-union* with himſelf, is plain, becauſe this is the ſingle Mean, to bring them to this End; for the known Law of Nature obtains even here, and every where, to wit, that *Action and Re-action is mutual*, ſo that if the *ſupreme Infinite* muſt draw them to him in Order to make them happy, they muſt have a Principle of being drawn towards him. That it muſt be a *radical* and *eſſential* Principle, is evident alſo; becauſe this Happineſs is the very End of their Creation, it being impoſſible infinite

finite Perfection ſhould make intelligent Beings, for any leſs or any other *End*. Wherefore the *ſupreme Infinite* could not make intelligent Creatures, without *implanting* in their Natures, a *moſt ardent Deſire*, an *eſſential* Principle, interwoven in the Subſtance of the ſpiritual Natures, of being re-united with himſelf, in order to make them as happy as their relative Natures will admit of. Beſides, intelligent Beings (as to their ſpiritual Part) are Images of the *ſupreme Infinite*, by *Prop*. 15. In him there is an *infinite Deſire*, and *Ardor* of poſſeſſing and enjoying himſelf, and his own infinite Perfections, in order to render him happy; he himſelf is the *ſole Object* of his own, and of the Felicity of all his Creatures: There muſt therefore be an Image of this his infinite Deſire after Happineſs in all his intelligent Creatures; and this Image can be nothing but this *Principle of Re-union*, ſince nothing but this can unite them with him, to make them happy. Laſtly, an intelligent Being, coming out of the Hands of *infinite Perfection*, with an Averſion, or even Indifferency, to be re-united with its *Author*, the Source of its utmoſt Felicity, is ſuch a Shock, and Deformity in the beautiful *Analogy of Things*, ſuch a Breach and Gap in the *harmonious Uniformity*, obſervable in all the Works of the Almighty, and that too, in the nobleſt and higheſt Part of his Works; as is not conſiſtent with finite Wiſdom and Perfection, much leſs with the *ſupremely infinite* Wiſdom of the ALL-PERFECT. Wherefore, *&c.* q. e. d.

Corollary. I.

HENCE we may diſcover the Source of *natural Conſcience,* and of all thoſe Motions and Convulſions, that are raiſed in the Breaſts of compounded intelligent Beings, upon the *Commiſſion* and *Omiſſion* of certain Actions: Of that *Comfort*, *Joy*, and *Support* in ſome; and of that *Dejection*, *Dread*, and *Terror* in the Minds of others; where no natural Cauſe can be aſſigned. Hence the *noble* and *ſublime Diſcoveries* of the *ancient Heathen Philoſophers,* in the Principles of moral Virtues, without the Aſſiſtance of Revelation. Hence it is, that *Scelerats* can by no Arts, nor any Amuſements how violent ſoever, ſtifle the Cries of a wounded Conſcience; and hence alſo, it is, that honeſt and upright Minds are ſometimes ſwallowed up by a Tranquillity and Peace that ſurpaſſes natural Underſtanding. That this *Principle of Re-union* is defaced, buried, and in ſome Meaſure as it were obliterated by contrary *Attractions*, by *Senſuality*, and the violent Amuſements of Licentiouſneſs in compounded intelligent Beings, is no more an Argument againſt its *eſſentially* belonging to intelligent Beings, than the *Ideotiſm* of ſome is an Argument againſt the Principle of Reaſon in human Nature. But,

Lemma II.

FROM this *Principle*'s being ſo *radically* implanted in all the Individuals of intelligent Beings, and from the ſo very few Inſtances and Remains of it, conſpicuous in the human Race; we may gather the infinite and univerſal Degeneracy and Corruption of this Set of intelligent Beings, from their *primitive* and *original* Inſtitution. This Principle was moſt certainly implanted in their Creation, in the very *Fund* and Subſtance of their Natures, and yet there remains but few Footſteps, and Inſtances of its *Being* or *Effects*. There are indeed (as in the greater World) ſome *legible Characters*, *ſtrong Out-lines*, and *prominent Lineaments* of its original Beauty, ſome *magnificent Ruins*, which ſhew what it had been, enough to demonſtrate the *original impreſſion*, Beauty, and real Being, of ſuch a *Principle*, in all the Individuals of this Race, as it is evident from the *Pangs* and *Tortures* of *natural Conſcience*, when it is counter-acted. But the little Effect it has, from what it was deſigned to have, is an evident Demonſtration of the deep and univerſal Corruption of this *Set* of intelligent Beings.

Corollary III.

HENCE we may farther diſcover the Force, Truth, and Univerſality of this wonderful *Analogy of Things*; whatever we diſcover of the Works of Nature, is from this Source, and whenever we get the leaſt *glympſe* into the Manner of the divine Operating, we diſcover freſh Inſtances of this *Analogy*. This *Principle of Re-union* in intelligent Beings, wonderfully *analogiſes* with that of *Attraction* in the material World: As to the *ſupreme Infinite*, it may be very properly called his *Attraction* of them, and as to them, their *central Tendency* or *Gravitation* (ſo to ſpeak) toward him; and this *Principle of Re-union*, if attended to, duly cultivated, and expanded, would as certainly bring about the temporal and eternal Happineſs of all intelligent Beings in the *ſpiritual World*; as that of *Attraction* brings about the comely and *harmonious* Motions of the *great Bodies* of the *material World*. This *Principle of Re-union* is the *original* Sourſe and *Spring* of the *Deſire*, afore-demonſtrated to be the *cardinal* Faculty of the ſpiritual Part of intelligent

Beings. The *Principle of Re-union* is the *Root* and Foundation of the *Deſire*, in regard to its firſt Cauſe and original Impreſſion by the *ſupreme Infinite*. This *Principle of Re-union*, as it is conſidered as an infinitely *active, quick,* and *ſenſible* Faculty, in the ſpiritual Part of compounded intelligent Beings, is the *Deſire of Happineſs*; as it is conſidered as a *theological* Virtue, it is *Charity*; and as it is conſidered as a *Rule* of Action, it is *natural Conſcience.* But the *Principle of Re-union,* in its whole Extent, as it was originally impreſſed upon, and quite interwoven with the intimate *Fund* and Subſtance of the ſpiritual Part of compounded intelligent Beings, is the *Source, Origin,* and *Root* of all theſe.

Corollary IV.

HENCE, the true and genuine Nature of *moral Good* and *Evil,* and of all the moral Virtues, and *ſocial* Duties of Life, as from their genuine Fountain and Source, is to be derived. Whatever retards, or oppoſes this *Re union* in intelligent Beings, is to them *moral Evil,* whatever promotes or advances this *Re union*, is to them *moral Good.* Beſides, this *Principle of Re-nnion* duly cultivated, regularly unfolded, and carefully attended to, muſt neceſſarily produce and perfect in the Soul *univerſal Charity*; that is, the Love of the ſupreme Being, and of all his Images in a due Subordination; and thereby inſtruct, and beget in the Soul, all the *moral* Virtues and *ſocial* Duties of Life. All theſe being *virtually* and neceſſarily included in *Charity*. But to explain this Matter (which is of the utmoſt Conſequence towards a right Apprehenſion of *ſolid Religion*) a little farther, I ſay

Propoſition XIX.

Charity, or the Love of the *ſupreme Being*, and of all his *Images* in a proper Subordination, according to their *Rank* in the *Scale* of Subſiſtences, is the neceſſary Effect of this *Principle of Re-union*, when fully expanded and ſet at freedom.

Demonſtrat. By *Corollary* 4. *Prop.* 17. nothing but the *ſupreme* and *abſolute Infinite* can adequately fill and ſuperabundantly ſatisfy the *infinite Deſires* of intelligent Beings; the *Deſire* in intelligent Beings is their *Love*; for no intelligent Being can deſire any Thing in order to make it happy, but what it *loves*; or can *love* any Thing, but what it *deſires* to enjoy, (*Indifference* being the middle State, and *Averſion* the equal Oppoſite, both to *Love* and *Deſire*.) So that an infinite *Deſire* of the *ſupreme* *and abſolute Infinite*, is an *ardent Love* of that *Being*. But the *Principle of Re-union*, expanded, ſet at freedom, and arrived at its *ultimate End* and *Center*, is intirely the ſame with the *infinite Deſire* in intelligent Creatures, poſſeſſed of its ſole and proper Object, the *ſupreme and abſolute Infinite*. Conſequently the *Principle of Re-union*, expanded, and ſet at freedom, in order to arrive at this ultimate End and Center, muſt *neceſſarily* beget in the Spirit of intelligent Creatures, this *infinite Deſire* of the ſole and proper Object; that is, an *infinite Love* of the *ſupreme Being*: And by neceſſary Conſequence, a *Love* of all his *Images* in Proportion to their *Reſemblance* of him; that is, the *Principle of Re-union* when expanded, and ſet at freedom, muſt neceſſarily beget in the Spirits of intelligent Beings, a *Love* of the *ſupreme Being*, and of all his *Images* in a proper Subordination, according to their *Rank* in the *Scale* of Subſiſtences; that is *Charity*. q. e. d.

Corollary I.

HENCE, *Charity*, or the *Love* of the *ſupreme Being*, and of all his *Images* in a proper Subordination, in its true and genuine Nature, is not founded on *Intereſt*, or the *Views* of *Rewards* and *Puniſhments*; but altogether on the *abſtracted Perfections* of its Object, the *ſupreme Infinite*. *Charity* in its Origin, and as it ought to be, according to the true *Analogy* of Things, is a *phyſical* and neceſſary Conſequence of the *Principle of Re-union*: It flows naturally from an *implanted* Faculty, and has for its Object the *ſupreme Infinite*, in his own *independent* and *eſſential* Nature, as he is *abſolutely* good and perfect, without any *collateral* Views or Regards. *Charity* is in a higher Degree, and in a more noble Creature, (one, to wit, endowed with Freedom and Liberty of acting,) what *Motion*, proceeding from the Principle of *Gravitation*, is in *Brute-matter*, or what the *Tendency* of the *Planets* is towards the *Sun*, *viz.* a natural Conſequence of an implanted Principle. This is ſo clear and ſo certain in true *Philoſophy*, that it is Matter of Aſtoniſhment to me, how it came once to be doubted, much leſs diſputed. *Henry Moor's* Argument on this Head is as cogent and juſt, as any Demonſtration in *Euclid* or *Apollonius*. " *As the Object of the Intel-* " *lect* (ſays he) *is that which is ſimply* " *true, and is aſſented to as ſuch, and not* " *as true to this particular Intellect which* " *contemplates it, ſo there is an Object that* " *is ſimply Good and Lovely, and to be lov-*

“ *ed as ſuch, without regard to the Party* “ *that thus loves it.*” And in another Place, he uſes the familiar Illuſtration of *Ginger-Bread* and *Mathematicks*, wherein he grants the former may be a Spur to the latter, 'till Age and good Senſe, with the Knowledge of the intrinſick Beauty and Worth of the latter, makes the *Student* in love with the *Study* it ſelf, without any Conſideration of the *childiſh Bait*. As in natural Love, Perſons become often enamoured of *outward Beauty*, without any particular Knowledge of its Poſſeſſor, or its *Attainableneſs* by them; ſo without all peradventure, *infinite Perfection*, for its own intrinſic *Pulchritude*, muſt be the proper Object of *divine Charity*, without any particular Regard to the Party loving it. Not that *Intereſt*, or a *View to Rewards* and *Puniſhments*, is not often the only Motive of *divine Love*; And always is a very proper and *laudable* one, when *infinite Perfection* is the Object; And in reality, it is as high as moſt of the *lapſed Race of Adam*, in their degenerate State, can riſe to: But as Things are in their *original Natures*, were in their *Integral*, and muſt be in their *redintegrated* State; *Infinite Beauty*, or Perfection, without any Regard to *Self-Intereſt*, or any View to *Rewards or Puniſhments*, is, was, and muſt be the proper and ſole Object of pure and perfect *divine Love* or *Charity*.

Corollary. II.

HENCE the *Service*, *Worſhip*, and *Homage*, we owe to the *ſupreme Being*, is founded intirely upon his own *original Excellencies*, and Perfections, and not on his *Rewards* and *Puniſhments*; there neither ever was, nor ever could be any room for *Contracts*, or *Pactions*, between the *ſupreme Being*, and his intelligent Creatures, in the original Conſtitution of Things. *He made all Things by the Word of his Power, and for his Pleaſure they are and were created. Infinite Perfection* is to be *loved*, *admired*, *adored*, and *ſerved*, for being infinitely perfect, *antecedently*, and without any Regard to Creatures: And when Creatures are brought into Exiſtence, this *primary* Reaſon of *Love*, and *Adoration* ſubſiſts, and in order of Nature and Dignity is *prior* and preferable to all other Reaſons. *Creation* adds nothing eſſential to *infinite Perfection*, but a *Circumſtance* only, which too intirely vaniſhes when brought into Compariſon with the original Beauty of the *abſolute Infinite*. All his Creatures, that act naturally, correſpond, and are faithful, to the greateſt Exactneſs, to his *original Impreſſions*, and his appointed ed and deſigned Ends on them. The *Celeſtial Orbs*, the *Sun, Planets, Comets*, and *fixed Stars*, thoſe huge unweildy Maſſes of Matter, revolve in regular *Periods* and conſtant Order, by their impreſſed Power of *Gravitation* and *primitive Frame*. The *vegetable Tribes* are faithful to his appointed Seaſons. The *Brute Animals*, ſacredly obey their *inbred Inſtincts*. Only his compounded intelligent Creatures, if they at all obey, do it for their own, not his Sake; that is, they themſelves are their own *ultimate End*, contrary to what was *Demonſtrated*, *Corollary* 4. *Prop*. 17. It is true in the Nature of Things, it is abſolutely impoſſible, to ſeparate the *Duty* and *Felicity* of intelligent Creatures, their *Happineſs* conſiſting in their being in the *Order* of their Nature, that is, of GOD *its Author*, and their only *Duty* being faithfully to obey this *Order*: Yet theſe two may be conſidered apart, and *mentally* abſtracted the one from the other: And the laſt muſt be *Anterior* to, and in the Order of Things, muſt preceed the former: So that when we invert this *Order*, and bring in our *Happineſs*, as the ſole *Motive*, or the preferable *Motive* to our *Duty*, we beome guilty of the moſt *groſs and blackeſt Idolatry*: For he who loves and worſhips the *ſupreme Being*, only becauſe his own *Happineſs* is thereby brought about, makes himſelf and his own Happineſs his *ultimate End*, and ſo places himſelf in the *Rank* and *Order* that belongs to GOD only, and himſelf becomes his own *Idol*. He only can be ſaid truly to *love* GOD, with a *Love* undivided, and worthy of him, who having *loved him* as far as *Rewards and Puniſhments* will carry him, goes on ſtill further to *love him*, loſes all views of theſe *mercenary Motives*, and ſeeks for no Fuel to feed the *celeſtial* Flame, but the *unexhauſtible Pulchritude*, and Perfections of the *beloved Object*.

Propoſition XX.

Charity, or the pure and diſintereſted *Love* of GOD, and of all his *Images* in a proper Subordination, is the *End of the Law*; the *Accompliſhment* of all the *Graces*, and the conſummate Perfection of *Chriſtianity*.

Demonſtrat. *On theſe Two*, to wit, *the Love of* GOD and *our Neighbour*, *hang both the Law and the Prophets*, ſays the ADORABLE AUTHOR OF CHRISTIANITY; and the *Apoſtle* ſays, *the End of the Law is Charity*. If the *ſupreme Being* is the *ultimate* Object of the Felicity of all intelligent Creatures, and *Charity* the Mean to attain this End, as is evident from the *preceeding Propoſition*; then is *Charity* the conſummate *Perfection of Chriſtianity*. The whole of *Chriſtianity* is nothing but Rules for attaining this *Love*, or Meaſures whereby to remove the Impediments that hinder this *Principle of Re-union* (the Source of *Charity*) from operating, or Means to deſtroy the contrary *Attractions* which diſturb the natural Operation of this *Principle of Re-union*; which would of it ſelf, if not ſtifled, oppoſed, and counteracted, neceſſarily beget this *Divine Charity*, whereby the Soul would inſtantly be united with its *Center*, and *ultimate End*, the *ſupreme and abſolute Infinite*. q. e. d.

Scholium.

To this Doctrine of *pure Love*, there are made but two Objections which have any Weight or Force in them. The firſt is, That *the Motives* for *Love and Obedience*, urged by *Moſes* and the *Prophets*, CHRIST and his *Apoſtles*, are founded on *Rewards and Puniſhments*; and that therefore without *Blaſphemy*, we are not to offer at, or pretend to, more high and ſublime *Motives* or Principles, than the *Friend of* GOD, and the SON OF GOD, (the Standard in their ſeveral Diſpenſations, of Purity and Perfection) thought fit to preſs or propoſe. The plain and genuine Anſwer to this Objection is, that the *Author of our Being*, *who loves all his Creatures better* *than they can love themſelves*, uſes all *Motives* that are *honeſt*, *laudable*, and *juſt*, to gain them; he knows perfectly the *Frame* and *original Complection* of all his Creatures, and that in their *lapſed State* they muſt aſcend to Perfection by Steps and Degrees: and conſequently that ſome are to be wrought upon by one *Motive*, others by another, and that generally the firſt Steps are mounted by the Force *of the Terrors of the Lord*, before *the Love of* GOD *is ſhed in their Hearts*, There are *Babes in* CHRST, as well as *grown and perfect Men*, and their Food (or Motives to Charity) muſt be as their Years and Strength are: But our SAVIOUR *tells us*, *we muſt love the* LORD *our* GOD, *with all our Heart*, *with all our Soul*, *with all our Strength*, *and with all our Mind*; and if ſo, we ſhall have very little *Love* left behind for *our ſelves*. And *his beloved Diſciple* tells us, *that perfect Love caſteth out Fear*, and conſequently *Hope*; that is *Rewards and Puniſhments*, which are true, good, and *ſalutary* Motions, though not the beſt. The ſecond Objection is from the Impoſſibility of *Loving* or begetting *Love* without a Regard to *Rewards or Puniſhments*. But this Objection ariſes from Ignorance of the true Nature of this Affection of the Soul. *Love* is the Complection of the *Will or Deſires*, as was ſhown *Prop.* 15. it belongs to the *unin-*

uninlightened Faculty of the Mind, the *Will*, and not to the enlightened Faculty, as the *Underſtanding* is; and ſo naturally, and of it ſelf has no real Reſpect to *Rewards or Puniſhments*, which are Motives offered by the Underſtanding: We *love*, becauſe we will *love*; without *Reaſoning*, or becauſe the Object of our *Love* is amiable, and not becauſe it will hurt or heal us. *Love* is blind, and belongs intirely to the *Will*, and not to the *Intellect*. But paſſing this, as perhaps too *metaphyſical*, I anſwer, *2dly*, As we may for one ſingle *Inſtant*, and for one ſingle *Act*, abſtract from a Reward, forget it, or counteract it, (which no Body, who knows the preſcindent Faculties of the Soul, and that *Love* and *Rewards* are eſſentially united in their own Natures, can deny) ſo we may thus *abſtract* again, and again, and ſo *in Infinitum*, and thus beget a *Habit*: For what may once be done, may for any Impoſſibility in the Nature of the Thing be done *for ever*: This is *Demonſtration*. But I proceed.

Propoſition XXI.

In all intelligent Beings, there muſt be *Faculties* fitted for all the ſeveral *Ranks* of Objects, in the *Univerſitas rerum*; that is, ſince there are evidently three *Ranks* of

Objects, in the *Univerſitas rerum*; to wit, the ***material Syſtem*** of Things, the *ſpiritual World* of created Spirits, and the *ſupreme and abſolute Infinite*; Intelligent Beings muſt neceſſarily be fitted with *Faculties* ſuited to theſe three generical Ranks of Objects.

Demonſtrat. This is evident from the ſeveral different *Mediums*, which all infer the ſame Concluſion. 1. Intelligent Beings are *Images* of the *ſupreme Infinite*, who alone perfectly comprehends and knows himſelf and all his Creatures, that is, all theſe three *generical Objects*, in the *Univerſitas rerum*. He has different Senſations and Perceptions (as far as *Diverſity* can be conſiſtent with his infinite *Simplicity*) ariſing in himſelf from all theſe three different Objects, and conſequently Faculties fitted for them: Therefore intelligent Creatures, his *Images*, the *Repreſentations* of all his communicable Perfections, muſt of neceſſity have *analogous* Faculties, fitted for all theſe three different Objects. 2. There are *Relations*, incumbent upon all intelligent Beings towards each other, and towards the *ſupreme Infinite*; ſuch as *Love*, and *Benevolence*. Therefore intelligent Beings muſt be endowed with Faculties fitted for receiving the Impreſſions, and to perceive the Effects of theſe *Relations*, elſe they would be in vain: And theſe are two of the different *Ranks* of

Objects; and no Body queſtions intelligent Beings, being fitted with Faculties for the third; to wit, the *material Syſtem* of Things. 3. As to the *ſpiritual World*, including the *ſupreme* and *abſolute Infinite* as its *Head*; The *Principle of Re-union*, whoſe Neceſſity in intelligent Beings, I have now demonſtrated, makes it abſolutely neceſſary, they ſhould be provided with *Faculties* fit for *Communication* and *Union* with the *ſupreme Infinite*; elſe, they were not ſuſceptible of that Happineſs which was the *ſole End* of their Being. 4. That which is the *apodeictick Demonſtration* of the Truth of this *Propoſition*, and at the ſame Time proves the Diverſity of theſe ſeveral Faculties, to be as real as that of the Objects, is the manner after which compounded intelligent Beings, are provided with *Faculties* for the *material Syſtem* of Things. Material Things are preſented to them only through their Senſes; they have a real and material *influx* on theſe, elſe they are not really perceived; and all real Knowledge of material Things is conveyed into the Underſtanding, through theſe Senſes: Wherefore it is evident, compounded intelligent Beings are endowed with a Faculty of perceiving or receiving material Things through their Senſes, which is called *Perception*. Next they have a Faculty of painting theſe Perceptions or their Images when the Objects are abſent, and this is called *Imagination*; and laſtly, a Faculty of *combining* and comparing the real Preceptions of theſe material Things, or their *Images*, and this Faculty is called *Reaſon:* And all theſe *three* diſtinct Operations belong to the *rational Soul*, in order to fit it for Communication with the *material* World. Now by the *Analogy of Things*, ſuch like, and *ſimilar* Faculties, muſt of Neceſſity belong to the *ſpiritual* Part of compounded intelligent Beings, to fit them for a Communication with the two remaining *Ranks* of Objects; to wit, a ſpiritual *Perception*, and *ſpiritual Senſes*, *Imagination*, and *Underſtanding*, for the *ſpiritual* World of intelligent Beings; and *divine Senſes*, *Perception*, *Imagination*, and *Underſtanding*, for communicating with the *ſupreme Infinite*. For this *Analogy* will perpetually hold good and true, from the *Simplicity* and *Unity* of the *divine Nature*; to wit, ſuch as the Faculties are by which we *communicate* with the *material* World; ſuch, *analogically* and with proper Limitations, are thoſe by which we *communicate* with the World of Spirits, and the *ſupreme Infinite*. And as we ſee compounded intelligent Beings have bodily *Senſes*, which ſolely belong to the *material* World, ſo in the *Analogy* of Things, they muſt have Faculties different and diſtinct in themſelves, peculiarly fitted and appropriated to the other Objects in the *Univerſitas rerum.* Wherefore, *&c.* q. e. d.

Corollary I.

HENCE we may conceive the Reaſon why, in *holy Writ*, the whole Man is diſtinguiſhed into *Body*, *Soul*, and *Spirit*; whence comes the Diſtinction of the *natural*, and *ſpiritual* or *inward Man*; between the *Law of the Members*, and the *Law of the Mind.* Theſe Diſtinctions and Diviſions, I ſay, are eaſily conceived from the foregoing *Propoſition.* For the Body and rational Soul belong to this *material Syſtem* of Things, and are fitted with Faculties for *communicating* with it, and is called the outward Man: And the following it's practical Dictates in Rebellion againſt, and in Oppoſition to the Dictates of the *Spirit*, the *inward Man*, the eſſential *Principle of Re-union*, the *Law of the Mind*, which is fitted only for *communicating* with the *ſupreme Infinite:* I ſay, the following the practical Dictates of the firſt, in Oppoſition to the *Dictates* of the latter, is called, in the Language of the *Spirit*, the following the *Law of the Members.*

Corollary. II.

As there are *three* different Principles, Orders, or *Predicaments* of Being; to wit, GOD the *Creator of all Things*, *created Spirits*, and *material Bodies*; ſo there are *analogically*, in compounded intelligent Beings, *three* different and diſtinct Principles, adapted and appropriated, for communicating with, and enjoying theſe reſpective Objects: Whereof every one is endowed with proper *Senſes*, *Powers*, and *Faculties*, different and diſtinct from each other; that is, as the *material Syſtem* of Things, is the proper Object of the Senſes, and rational Soul; and as this Principle is fitted with outward *Senſes*, *Perception*, *Imagination*, *Underſtanding*, and *Will*; So the Principle, whereby they are fitted to communicate with the created ſpiritual World, is endowed with inward *Senſes*, *Imagination*, *Underſtanding*, and *Will*; and the *ſupreme Spirit*, or third Principle, whereby they are fitted to communicate with the ſupreme uncreated Infinite, is endowed with *inmoſt* (ſo to ſpeak) *Senſes*, *Imagination*, *Underſtanding* and *Will*; and all theſe in their *primitive* and *original* Conſtitution, in Subordination, *Harmony*, and Agreement, without Contrariety or Confuſion one with another. This *Corollary* is as certain as the *Analogy* of Things is.

Corollary III.

HENCE we may deduce the true and genuine Nature, and Extent, of the *Degeneracy*, *Corruption*, and *Fall* of the human Race of intelligent Creatures, which conſiſts in the *Confuſion*, *Diſcord*, *Rebellion*, and *Contrariety* of theſe different and diſtinct Principles one with, and againſt another; in throwing off that *due Subordination*, *Subjection*, and proper *Rank* and Order, that was originally eſtabliſhed among theſe Faculties; according to the Dignity of theſe different Objects, the Order of Nature, and the *Analogy* of Things; and following the practical *Dictates*, and Concluſions deduced from theſe rebellious Faculties: That is, when in this *anarchical* and rebellious State of human Nature, the Faculties belonging to the material World preſume to judge of, and determine the Nature of the Subjects belonging to the *ſupreme* Spirit; take the Government and Adminiſtration of the whole Man, which properly belongs, in the Order of Nature, to this third Principle; lead the other Principles as *Slaves* and *Captives*, and force them to comply with the practical *Dictates* they preſcribe, and deduce in their uſurped Superiority; and under this compliance, begetting and producing phyſical and durable Effects, the whole Order of Nature, and the material *Syſtem* of Things, ſo far as theſe *phyſical* and durable Effects reach, become diſtorted, inverted, and corrupted.

Propoſition XXII.

THE *rational Soul* is not that Faculty in compounded intelligent Beings, which in the Order of Nature, and the *Analogy* of Things, is appropriated for the ſpiritual World; (including the ſupreme Infinite at it's Head.)

Demonſtrat. Though this be a neceſſary *Corollary* from the preceeding *Propoſition*, yet ſince it is of great Conſequence to true divine Knowledge, to have it's Truth eſtabliſhed beyond all poſſibility of *Cavil*; I ſhall here ſuggeſt ſome other *Mediums*, or ſet thoſe already ſuggeſted in another Light, from whence the ſame *Concluſion* may be deduced. And 1. This is evident from the Nature of this *Faculty*, and the manner of its operating. *Reaſoning* is the comparing, or the conſidering the *congruity*, or *incongruity* of the Perceptions, ſuggeſted by the Senſes, or of the *Idea's* lodged in the Memory, or painted on the Imagination, to one another; and *Reaſon* is the Faculty whereby this is performed. Now the *Senſes* ſend in only the *Influxes* of material Things, and the *Imagination* and *Memory* preſent only their Pictures or *Images*, when the Objects themſelves are abſent; and here is all the *rational Soul* can do. But nothing of theſe belong to the *ſupreme* and *increated Infinite*, nor the *ſpiritual World*. 2. It is acknowledged by all, and every one's Experience demonſtrates it to him, that the *rational Soul* is fitted for *communicating* with the *material World*. Now ſince Body and Spirit are preciſe and proper Oppoſites, it would be as *diſſonant* and *incongruous* in the *Analogy* of Things, that the ſame Faculty ſhould be the Principle of Communication with theſe two Objects ſo widely diſtant, as that the *Eye* ſhould both *hear* and *ſee*. It is true, it may be ſaid, that the *rational Soul* might have been originally endowed with ſuch *Energy* and Capacity, as to be fitted for *communicating* with *both Worlds*; but this is meerly *gratis Dictum*, and perfectly contrary to the *Analogy* of Things; it being impoſſible to bring an Inſtance of Nature, where Things ſo widely diſtant, and preciſely oppoſite, are received by *one and the ſame* Faculty. We ſee, in the *Body* or loweſt part of the human Compoſition, that it is inſtructed with *Organs* fitted for all the poſſible Ways, material Things can act upon it; and they are not near ſo widely diſtant and different in the manner of operating, as Bodies and Spirits in their Natures are. 3. There is a Two-fold Knowledge of material Things, one *Real*, when the Thing it ſelf, and the *real* Action and Impreſſion thereof on our Senſes is perceived: The other *Ideal*, when the *Image* or *Idea* of a Thing abſent in it ſelf, is repreſented to, and conſidered on the *Imagination*: For Inſtance, the *Heat*, *Light*, and *chearful Influences* of the *Sun* ſhining on us, are widely different from the View and Conſideration of its *Image*, or *Idea* on our Fancy. In the *Analogy* of Things, and according to the conſtant Order of *Nature*, ſuch muſt our Knowledge of *ſpiritual* Objects be; to wit, the one *Real*, when the Objects make a real Impreſſion upon the *appropriated* Faculty; the other *Ideal*, when we frame a Notion of its abſent Subſtance and Qualities. Now it is very plain, the *rational Soul* is not fitted for this firſt kind of Knowledge or Perception of *ſpiritual* Objects, ſince a great many *philoſophical* and *learned* Men, who have exerciſed this Faculty in its greateſt Strength and *Vigour*, have denied the Exiſtence of ſuch Objects. 4. Laſtly, That moſt certain and ſelf-evident *metaphyſical Axiom*; to wit, *that nothing can be in the Underſtanding, that* *was not firſt in the Senſes*, is a certain *Demonſtration*, that the *rational Soul* is not the Faculty in intelligent Beings, *appropriated* to the *ſpiritual* World; for every Body allows, that *ſpiritual* Beings, *as ſuch*, can never be conveyed through the bodily Senſes, to the Underſtanding: And therefore we muſt either be intirely deprived of Faculties, for communicating with *ſpiritual* Beings (that is, we muſt be deprived of the only Means of our *ſupreme Felicity*, and for attaining the End, for which alone we were created; to wit, communicating with the *ſupreme and abſolute infinite Spirit*.) Or elſe, we muſt be endowed with Faculties diſtinct from the *rational Soul* for that purpoſe: Upon all which Accounts, it is evident beyond the moſt remote poſſibility of Doubt, that the *rational Soul* is not that Faculty in compounded intelligent Beings, which in the Order of Nature, and the *Analogy of Things*, is *appropriated* for the *ſpiritual* World: But that they are endowed with a *rational Soul*, and bodily Senſes, to communicate with the *material* World, and with a *Spirit* and *divine Senſes*, to communicate with the *ſupreme Infinite*: So that the mentioned *metaphyſical Axiom* continues juſt and true, as it ſhould according to the *Analogy of Things*: That as *material* Things are conveyed to the *rational Soul*, through the *bodily Senſes*, ſo *ſpiritual* Things (thoſe that relate to the *ſupreme Infinite*) are conveyed through the *divine Senſes*, to the *Spirit*. q. e. d.

Corollary I.

HENCE, in the *Analogy of Things*, as the *Light* of the *Sun* (that noble and glorious *Repreſentation*, *Image*, and *Vicegerent* of the *ſupreme Infinite*, in the *material* World) is the *Medium*, through which *material* Things are ſeen and perceived in our *Syſtem*; ſo the *eſſential Light* of the *ſupreme Infinite* himſelf is the ſole *Medium*, by and through which, his Nature and infinite Perfections are to be underſtood, and comprehended: And therefore, as certainly as the *Sun* ſends forth his Light on the whole *material* World, without Bounds or Limits, on *the Juſt, and on the Unjuſt*; ſo certainly, the *Sun of Righteouſneſs*, the *Pattern* and *Archetype* of our *material Sun*, ſends forth his enlightening and enlivening Beams on all the *Syſtem* of created intelligent Beings; and is, *that Light which enlightens every Man that cometh into the World.*

Corollary II.

HENCE it is evident, that he who would judge, determine, and purſue the practical

Concluſions of theſe Determinations, about the Nature and Properties of *ſpiritual* and *divine Things*, by his *Reaſon*, would act as incongruouſly, and contrary to the *Analogy of Things*, as he who would taſte Colours (as ſuch,) and look into Sounds. The higheſt that this *Faculty* can juſtly and congruouſly pretend to in theſe Matters, is from the known, certain, and experienced Nature, and Properties of *material* Things, (to which the *rational Faculty* is in ſome Meaſure adequate) by a proper *Analogy:* And from the *Viſibles* being low *Images* of the *inviſible*, and *ſpiritual*; to frame *ſimilar*, but imperfect *Likeneſſes*, and Repreſentations of theſe ſuperior Objects, their Natures and Properties, as we have endeavoured to do in the preceeding *Propoſitions*; and this really, and in fact, is all that *Reaſon* can do in theſe ſublime Matters.

Corollary III.

HENCE, we may diſcover the Errors and Impieties of *Spinoſa* and *Hobbes*; and the Miſtakes of a *later Philoſopher*, I mean (the otherwiſe *ingenious*) Mr. *Locke*. The *firſt* of theſe conſidered this *univerſal Syſtem of Things*, as a kind of a *Huge-brute-animal*, actuated by a fatal, neceſſary, unintelligent, undeſigning Principle; without either Wiſdom or Choice. The *ſecond* conſidered *human Nature*, (not as it really is, in its preſent State of *Probation* and *Purification*, a Mixture of moral and natural Good and Evil, but) in its *diabolical* and *reprobated* Eſtate: Not as groaning under its preſent State of Corruption, and waiting and panting *for the glorious Liberty of the Sons and Children of God*; but as it will be in an habitual confirmed Eſtate of the *Anarchy* and Rebellion of its Faculties one againſt another. In a Word, he conſidered *human Nature* only, as it is in the worſt of Men; or as it is ſuppoſed to be at laſt in a State of *final Impenitence*, and hardened Impiety; and this he took as his *Original* to copy after, and his *Model*, whereupon he was to frame his *human Creature*: And it muſt be allowed he has wrought it up to *the Life*. The *third* conſidered *Man* and his *Faculties*, not indeed in their already *reprobated* and *helliſh Eſtate*; but as he really now is, in the World, a Compoſition of moral and natural Good and Evil: And this State he has very fairly and juſtly repreſented ſo far as it goes. But then, either having no Notion, or at leaſt no Regard to his *higher Faculties* (which in natural and lapſed Man lye buried under the Rubbiſh of his preſent Corruptions and Senſuality;) nor to his *regenerated*, *redintegrated* and *re-eſtabliſhed Eſtate*, (to which he muſt be reſtored, before he can reach the *End* of his *Being*:) No, not ſo much as to conſider Man as he really is, a *fallen, depraved, vitiated* Creature, (in which State his *lower*, his *rational* Faculties are impaired; his *higher* Faculties in ſome meaſure *obliterated*, at leaſt buried and oppreſſed by the Load of preſent Corruption and Senſuality; and all of them in a State of *Anarchy*, Rebellion, and contrariety one to another:) I ſay, from having no Regard to thoſe *other* different *real* Eſtates of *human Nature*, his Accounts of its Faculties are *lame* and *imperfect*. His *Principles*, when applied (by himſelf or his Diſciples) to Subjects (to which Faculties are appropriated, higher than thoſe he *elicits* out of the meer lapſed State of human Nature,) of a more elevated Order, (ſuch as *Chriſtianity* and its *holy Myſteries, Faith, Grace, divine Revelation*, and *Inſpiration*, and the Means of Man's Recovery) debaſe theſe into *meer heatheniſh Morals*, or *human Philoſophy*, and ſink the *Oeconomy* of *the whole Wiſdom of the God-head*, even below the poor Contrivances and barren Speculations of many of the *gentile Sophiſts*. But thoſe who fully underſtand the *Principles*, and are convinced of the *Truth* of the *Propoſitions* I have laid down, will eaſily perceive the Ground of the Errors and Miſtakes of theſe three

Setts of *Philoſophers*, and be able to anſwer their Arguments without my being obliged to detain them.

Propoſition XXIII.

In the *Analogy of Things*, and Order of Nature, as the *material World* is to *univerſal Space*, its higheſt Limit and Boundary; ſo is the *ſpiritual World* to the *ſupreme and abſolute Infinite*, the higheſt Limit and Boundary of all Things.

Demonſtrat. This is evident from *Lemma* 1. with *Prop.* 9. and 17. There is a beautiful *Analogy* and Uniformity running through the whole *Syſtem* of Creatures. The *Viſible* and the *Created* are *Images* of the *Inviſible* and of the *Increated.* The *Syſtem* of intelligent Beings are more exalted, more noble, and more immediate *Images* of the *ſupreme Infinite.* The *Analogy of Things* runs quite through the whole *Syſtem* of Creatures, up to their original Pattern and *Archetype* in the divine Nature, in a continued Subordination and Scale, according to their reſpective Natures. The *material* World is an *Image* of the *ſpiritual* World, as the *ſpiritual* World is of the *ſupreme Infinite.* As *infinite Space* is the *Locus* and Boundary of the *material* World; ſo is the *ſupreme Infinite* the *Analogical Locus* (in whom *they all live, move, and have their Being*;) and the *Omega* of all Things, *ſpiritual* and *material*. And as *Space* is *ſimilar* to a *ſpiritual* Subſtance, ſo is that to the *divine Subſtance*, therefore, *&c.* q. e. d.

Corollary I.

HENCE *material* and *ſpiritual* Subſtances are both of them *extended*; for ſince the *material* World is to *univerſal Space*, as the *ſpiritual* World is to the *ſupreme Infinite*; and ſince both *Matter* and *Space* are extended, ſo alſo muſt *ſpiritual* Subſtances be: And the *divine Ubiquity*, and *Omnipreſence*, not *virtually* only, but *ſubſtantially* and *eſſentially*, makes it not unlikely that there may be, in the *divine Subſtance*, a Reſemblance of *Extenſion*, (ſo far as a Reſemblance and Similitude of Subſtances can reach, between a *relative Infinite*, ſuch as *univerſal Space* is, and the *ſupreme Infinite*,) but infinitely more pure and perfect, than that of created *Space* is, or can be.

Corollary II.

HENCE *Matter* and *Spirit* are Oppoſites in every other Quality, except in that of *Extenſion*; for ſince by *Converſion of Ratio's*, the *material* World is to the *ſpiritual* World, as *infinite Space* is to the *ſupreme Infinite;* and ſince theſe two laſt, to wit, *univerſal Space*, and the *ſupreme Infinite*, are Oppoſites in every other Quality, but in a Reſemblance of Subſtances, and that too, at an *abſolutely infinite* Diſtance, as is evident at firſt View; therefore the *other two* muſt be oppoſite in every other Quality but *Extenſion.* For though *extended Matter* be diviſible by being *extended,* yet *Space* is not actually to be divided, or one part of it ſeparated from another: Since it is the *univerſal Locus* of, and penetrates all Bodies: And it is in *this* Senſe, that the Oppoſition of theſe two Qualities in *Body* and *Spirit* is meant here.

Propoſition XXIV.

By the *Analogy of Things*, and according to the Order of Nature, a *Spirit* is an *extended, penetrable, active, indiviſible, intelligent* Subſtance.

Demonſtrat. By *Def.* 2. *Matter* is an *extended, impenetrable, paſſive, unintelligent, diviſible* Subſtance: And ſince by the *preceeding Corollary*, *Matter* and *Spirit* are in every other Quality oppoſite, except in that of *Extenſion;* therefore in Place of all the Qualities in the *Definition of Matter*, muſt be put their Oppoſites, excepting that of *extended*

tended Subſtance; (for *Extenſion* muſt imply a Subject;) and then a *Spirit* will become according to the *Propoſition*, an *extended, penetrable, active, indiviſible, intelligent* Subſtance. I have choſen the Word *Intelligent* in this, and its oppoſite *Unintelligent* in the other Definition; to wit, in that of *Body*, rather than that of *Thinking*; becauſe *Intelligence* is the Source and Principle of *Thinking*, and expreſſes the *Whole* of all the Faculties of *ſpiritual* Subſtances.

Corollary I.

HENCE, in Reſpect to their Subſtances only, a *material* Subſtance is an infinitely condenſed or incraſſated *ſpiritual* Subſtance: And on the other Hand, a *ſpiritual* Subſtance is an infinitely rarified or refined material Subſtance. As we have it in Holy Writ, there is a *natural* (or material) *Body*, and *there is a ſpiritual* and *a glorified Body*. For ſince *Matter* and *Spirit* have the Foundation of their Qualities common to both, to wit, an *extended* Subſtance: Since all their other Qualities are the one, reſpectively, the Oppoſite or Negative of the other. Since *rarifying* any Quality in *Body* and *Spirit*, is ſubtracting from its Intenſion and Energy; and therefore an infinite Rarefaction of a Quality is ſubtracting it intirely; and there being no *Mean* between *Penetrability* and *Impenetrability*, between *Paſſivity* and *Activity*, *Diviſibility* and *Indiviſibility*, *Intelligence* and *Unintelligence*, they being contrary and oppoſite; therefore the infinite Rarefaction of the one Quality is the Poſition of its contrary; for though the finite *Subtraction* of a *Negative* Quality (admitting no *Mean*) puts nothing, yet the infinite *Subtraction* of ſuch a *Negative* Quality puts the *Affirmative* (thus $-02 \times -0 = +0$. but $-\infty 0 + -0 = +0$;) therefore in *Matter*, *ſubtracting infinitely*, all its *poſitive* Qualities, or which is the ſame, (by the *preceeding Corollary*) in *Matter*, *ſubtracting infinitely* the *Negative* Qualities of *Spirit*, and then a *material* Subſtance will become a *ſpiritual* Subſtance; but ſince an infinite Rarefaction of a contrary Quality is the ſame with an *infinite Subtraction* of that contrary Quality, and ſince the ſame manner of Reaſoning holds good in an *infinite Condenſation* of the Qualities in *Spirit*: Therefore in Reſpect of their *Subſtances* only a *material* Subſtance, *&c.* q. e. d. The Caſe in ſhort, as I conceive it, is thus: A *ſpiritual* Subſtance, when infinitely condenſed and *incraſſated*, loſes its Qualities of *Penetrability*, *Activity*, *Indiviſibility*, and *Intelligence*; theſe being locked up, and as it were *crampt*, in this *Condenſation* of their *Subſtratum* (or the Subſtance in which they eſſentially inhered,) thus *infinitely compreſſed*; and the *Actuality* (as the *Metaphyſicians* ſpeak) of theſe *ſpiritual* Qualities being thus ſhut up and impriſoned, though their *Potentiality* be not quite deſtroyed, a *Craſs extended, impenetrable, paſſive, diviſible, unintelligent* Subſtance is generated, which we call *Matter:* But when this *Matter* thus formed of a *ſpiritual* Subſtance, is again *infinitely* refined and exalted; theſe *Powers* and *Qualities* are unlooſed, and ſet at freedom again exert themſelves as formerly, and thus become what they were originally made. But we muſt take care not to imagine that any *finite Subtilization, Diviſion, Refinement*, or *Exaltation* of groſs *Brute matter*, can in any the leaſt Degree, by any finite created Powers whatſoever, bring it to any but an infinitely diſtant Approach to this State of *Spirituality*; ſince it has been demonſtrated, *in the firſt Chapter of this Part*, that no Finite, how great ſoever, can be any finite Part of any *relative Infinite*, how ſmall ſoever: No Power leſs than his, *who out of the very Stones could raiſe* (by Virtue of their original *Potentiality*) *Children to Abraham*, can, out of *material*, bring *ſpiritual* Subſtances; or on the contrary, convert theſe into thoſe.

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Corollary II.

SINCE in aſcending from *material* Subſtances, there can be no ſuch Thing, by the *Analogy* of Things, and Order of Nature, as a *Jump* or *Leap*, from one Extreme to another, without paſſing through the intermediate Steps; and ſince in *material* Things, there are Subſtances of all Degrees of *Denſity* and *Rarity*; *Earth* being more denſe than *Water*, *Water* than *Air*, *Air* than *Ether*, *Ether* than *Light*; therefore in the *ſpiritual* World, there muſt be *Spirits* of all Degrees of *Rarity*; the one *Set* and Rank more pure and refined than the other in a perpetual *Scale*, 'till they aſcend ſo near the *ſupreme Infinite*, as Creatures can approach their *Creator*, or *Finites* the *abſolute Infinite*. And by the ſame *Analogy of Things*, as in the *material* World, theſe ſeveral Orders of Bodies, *Earth*, *Water*, *Air*, *Ether*, and *Light*, have their proper *Places*, *Elements*, and *Centers* when they reſt, and whither they tend, and out of which they cannot be detained but by *Violence*; ſo in the *ſpiritual* World, there are *Centers*, *Spheres*, and *Elements* of ſeveral Orders of *Spirits*, the one more pure and refined than the other, the more pure, ſtill penetrating the leſs pure, where they reſt and continue, to which, by their *ſpecifick* Degree of Purity they are confined, out of which they cannot be detained but by *Violence*; all in a Subordination one to another, depending on their particular Degrees of Purity, penetrating one another without Confuſion or Contrariety, 'till they arrive as near as poſſible to the *ſupreme Infinite*, who penetrates the whole *Syſtem* of Creatures.

Corollary III.

SINCE by *Prop.* 15. the *Deſires* are the *cardinal* Faculty of intelligent Beings, infinitely *active* and powerful, belonging to that Principle in them, which is appropriated to the *ſpiritual* World; by theſe *Deſires* therefore, thoſe of the ſame *Element*, and ſame Degree of Purity, are enabled to communicate one with another: But ſince the more pure, penetrate the leſs pure; but not *vice verſa* (as is plain from *Corollary* 1. of the *preceeding Propoſition,*) the more pure may penetrate the leſs pure, contrary to their *Deſires*; but not *vice verſa*; this Superiority of the more pure, over the leſs pure, being a neceſſary Conſequence of the greater Degree of Purity and Perfection. For as in the *material* World, the *Sun* purifies and rarifies terreſtrial Bodies, the nearer they approach, or are brought to him; and at laſt converts them into his Subſtance: So in the *ſpiritual* World, the *Sun*'s Pattern and *Archetype*, the *Sun of Righteouſneſs*, renders thoſe *Spirits* the more pure, penetrating, and as it were *deifies* their intimate Subſtances, whoſe *Elements* or Region is neareſt him; by which they more nearly partake of his Nature, who penetrates the whole *Syſtem* of Beings.

Corollary IV.

HENCE, ſince the *ſupreme Spirit*, in compounded intelligent Beings, is more pure (as being the higheſt Principle, and that *Breath of Life*, which they had immediately from the divine Subſtance) than the *rational Soul*; the *firſt* penetrates the *latter*, and the *latter* is but the *Medium*, and is it ſelf but of an intermediate Nature, between the two *Extremes*, *Body* and *Spirit*, coupling them together by its intermediate Subſtance: And in their primitive Order and Inſtitution, they were in a due Subordination one to another: The *Body* to the *rational Soul*, and *both* to the *ſupreme Spirit*; and in this Subordination, preſerved and maintained, and diligently cultivated, according to their reſpective Dignity; in the *Wills* obeying and approving of, and in the *whole compound's* purſuing the practical Inferences de-

duced

duced in this *State*, did the *original Rectitude* of theſe compounded intelligent Beings conſiſt. And in the Contrariety, Contradiction and Rebellion of theſe different *Principles* one againſt another; in the *Will's* obeying and approving of, and the *whole Compound's* purſuing the practical Concluſions deduced under this State of *Anarchy*; does the *Fall*, *Lapſe*, and *Degeneracy* of this *Set* of intelligent Beings conſiſt. And for the reſtoring, rectifying, and re-eſtabliſhing the Primitive Order and State of theſe three Principles, to an habitual and laſting Subordination, was the Incarnation of the *divine Logos*, and whole *Oeconomy* of the Redemption of Man.

Corollary V.

HENCE, the *ſupreme Spirit* may be dark, dead, and almoſt quite obliterated, as to its *ouvert* Acts (the *Principle* it ſelf being eſſential to, and interwoven with the moſt intimate Natures of all intelligent Beings,) when the *rational Soul* is full of *Idea's*, Pictures, and Images of Things. And on the other Hand, the *ſupreme Spirit* may be full of *Light*, *Brightneſs*, *ſubſtantial Knowledge*, *Joy*, and *Peace*, when the *rational Soul* is but weak, faint, and languid, and almoſt void of all *Idea's* and Images; theſe being not only ſeparable, but at laſt to be *actually* ſeparated, (ſo far at leaſt, as the Acts of the *rational Soul* can affect the *ſupreme Spirit*,) when the LOGOS (that *eternal and eſſential Word of God*) *which is quick and powerful, and ſharper than a two-edged Sword, ſhall pierce, to the dividing aſunder the Soul and Spirit.*

SCHOLIUM GENERALE.

THUS I have endeavoured to give ſome faint and imperfect Images, of the higheſt and moſt ſublime Speculations of *Religion* and its *Philoſophy*, in the *preceeding Propoſitions* and *Corollaries*. And though I am very far from thinking they are even juſt and compleat Images, and ſuch as might be drawn from the ſame Principles, by a more skilful Hand: Yet I am fully convinced, the *Propoſitions* and *Corollaries* themſelves are true and juſt, as to their Subſtance, whatever may be in my Way of explaining or demonſtrating them; ſo far at leaſt, as *Reaſon* can find out the *Truth* in ſuch *ſublime Myſteries*. I am alſo well ſatisfied that *Reaſon* can with any Propriety of Juſtneſs apply it ſelf to Objects, to which it is not *adequate* and appropriated, after no other Manner, but by ſuppoſing thoſe Objects, to which it is *adequate*, *Ima-*

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ges or Repreſentations of thoſe other Objects, to which it is not *adequate*; there being no other poſſible way for *Reaſon* to find a *Medium* of its Knowledge of Objects, that are conveyed to the rational Underſtanding, by none of the bodily Senſes, (as the proper Objects of its Faculties are.) And there being an abſolute Neceſſity from the *Simplicity* and *Uniformity* of the *divine Nature*, and of his manner of Operating, that all his Works ſhould be Reſemblances and *Images* one of another, (more or leſs perfect, according to their reſpective Natures) and alſo of himſelf, their *original Pattern* and *Archetype*: This Manner of Reaſoning, and this *Medium* of rational Knowledge, duly inſtituted, muſt be juſt and true, as far as it reaches. And thus far *Reaſon* can go, and not one ſtep further in the Knowledge of ſuperior Objects; it can frame and form *Images* of theſe ſuperior Objects, from what it finds and certainly knows of the *material* World, to which it is in ſome meaſure *adequate*. *Images*, I ſay, not *metaphorical* only, but *real* and *phyſical*, as a Statue repreſents a Man, a Picture in *Miniature*, one from the Life, as a *Seed* (which is really the Plant it ſelf in little) does a grown Plant, or an *Embryo*, the adult Animal: This is the Boundary of *Reaſon* in theſe ſuperior Objects. And it is very obſervable, that there are various Images in Nature, and in the *intellectual Species* of Things, (framed upon what the Senſes have already conveyed into the rational Underſtanding) of all the moſt unconceivable, the moſt abſtruſe and ſublime *Myſteries* of Religion and its Philoſophy, each ſuperior to another. Can there be a more perfect, noble, or lively Image in this lower World, of the *divine Nature*, *Light*, *Benignity*, *Greatneſs*, and *Power* required, than that of the *Sun*, in reſpect of our *planetary Syſtem*. His Beams ſhine, and are tranſmitted through all the *planetary* and *cometary* Regions, even into the *Syſtems* of the *fixed Stars*. He *attracts* all the *Planets* and *Comets* in our *Syſtem*, and is the Source of all their regular, uniform, and conſtant Motions and Influences. He warms, cheers, enlivens, and fertiliſes all the *Elements*, *Vegetables*, and *Animals*; and is indeed the *material Deity* of this inferior World. Is there not a plain and obvious Image of the EVER BLESSED-TRINITY IN UNITY, in every Order of Creatures; In the three Dimenſions of Bodies? In Nature's never riſing above the *third* Dimenſion in her regular Operations, as was ſhown in *Scholium*, *Prop.* 12? In the *three* infinite Powers of *univerſal Space?* In the *three* generical Diviſions of Objects? *Matter*, *created Spirit*,

Spirit, and the *ſupreme Infinite* in the *three* Diſtinctions and *univerſal Principles*, that comprehend the Whole of material intelligent Beings? The *Faculty*, *Object*, and the *Congruity* or *Incongruity* between theſe? The *Profane* and *Ignorant* may make a Jeſt of this *ternary* Chain, and aſcribe it to Chance or Fortune: But the *Analogy of Things*, and the regular Uniformity in Nature, make it evident to a *Demonſtration*, that it muſt have had its Riſe in its *original Pattern* and *Archetype*, the *divine Nature.* Even the eternal *Generation* of the *ſecond* Principle in the *God-head* of the *firſt*, and the eternal *Proceſſion* of the *third* Principle from the *firſt* and *ſecond*, comes evidently out of this *Analogical Ternary*, when elevated to its Origin in the divine Nature; as was ſhewn in *Scholium* 2. *Prop.* 17. And the Impoſſibility of increaſing or multiplying the divine and *ſupreme Infinitude*, even by it ſelf, ſo far as that it is uncapable of Increaſe, or Diminution; as was ſhewn in *Corollary* 2. *Prop.* 12. pictures forth the *Unity* of the divine Nature, in theſe *three* Relations of the *Whole* to the *Whole.* How noble a Repreſentation, in created Things, is the *univerſal Space* of the *divine Ubiquity*, *Infinitude*, and *ſpiritual Nature?* How lively a Picture, in the *intellectual Species* of Things, of *Creation*, or of God's producing the Things that be, out of the Things that were not, is that *Propoſition* demonſtrated *Corol.* 2 *Prop.* 2. $\frac{1}{1+1+1+1+1+1\,\&c.} \times 0 = \infty \times 0 = 1$. or $\overline{\odot+\odot+\odot+\odot+\odot\,\&c.} \times 0 = \infty\ \odot \times 0 = \odot$? The Production of a *Plant* from it's *Miniature* in the Seed, and of an *Animal* from an *Animalcule*, is an aſtoniſhing Repreſentation of the Reſurrection of the Body. Theſe and many ſuch Reſemblances, and Images in Nature, in the *ſenſible* and *viſible* Things, and in the *intellectual Species* of Things, derived through the Senſes, might be brought to illuſtrate and confirm the greateſt Difficulties and moſt abſtruſe Myſteries of *Religion*, and its *Philoſophy*. So certain and univerſal is the *beautiful Analogy of Things*, and ſo careful has the *kind* and *bountiful Author* of our Beings been, to ſupply us with Evidences in our *lower Faculties*, and *lapſed* Eſtate, of *thoſe Truths* he requires us to believe and receive. The *full* and *compleat* Conception and Knowledge of which belong not but to our *ſuperior* Faculties, and to our *reſtored* and *re eſtabliſhed* Eſtate. May we then uſe the Aſſiſtances, *Reaſon*, and the *divine Beauty*, has afforded us, (I may almoſt ſay, even beyond, and out of their natural Order) for increaſing our *Faith*, cultivating and expanding our *ſuperior Faculties*, reſcuing us out of our *Degeneracy* and

Corruption, *into the glorious Liberty of the Sons and Children of God*; and then, *in his Light we ſhall ſee Light.* Amen.

CHAP. III.

Of the USE of the Arithmetick for Infinites.

§ I. IN arithmetical Progreſſion; in which, let a denote the firſt, v the laſt Term, x the Difference, t the Number of Terms, and z the Sum of all the Terms, which in an aſcending Progreſſion are a, $a+x$, $a+2x$, $a+3x$, *&c.* but in a deſcending Progreſſion a, $a-x$, $a-2x$, $a-3x$, *&c.*

Having any three of theſe five, a, v, x, t, z. you may find the other two by help of theſe two Lemma's.

Lem. 1. $v=a+tx-x$ but $v=a-tx+x$.
Lem. 2. $2z=t\times\overline{a+x}$ when it deſcends.

I ſhall here only treat of aſcending Progreſſions, judging thoſe that deſcend as not properly belonging to the Arithmetick of Infinites.

Now in aſcending Progreſſions, both t and v are infinite.

Therefore *Corol.* 1. $v=tx$ in the Caſe of
And *Corol.* 2. $2z=tv$ Infinites.

Becauſe Infinites are not all equal, but vary in their Ratio's to one another, as much as finite Quantities do; therefore to avoid Confuſion, I ſuppoſe $v = \infty \times 1 = 1 + 1 + 1 + 1 + 1 + 1$ *&c.*

So from *Corol.* 1. $t = \frac{\infty}{x} = \infty \times \frac{1}{x} = \frac{1}{x} + \frac{1}{x} + \frac{1}{x} + \frac{1}{x} + \frac{1}{x} + \frac{1}{x}$ *&c.*

Prop. 1. $z = \frac{\infty^2}{2x}$. This follows from ſubſtituting the Value of t, *viz.* $\frac{\infty}{x}$ in *Corol* 2.

Examp. 1. Let $x = 1$. then $1 + 2 + 3 + 4 + 5$ *&c.* $= z = \frac{\infty^2}{2}$, that is, the Sum of all the natural Numbers continued in *infinitum* is equal to half the Square of infinite: And here it is to be noted, that in this Caſe only $t = v = \infty \times 1 = 1 + 1 + 1 + 1 + 1$, *&c.*

Examp. 2. Let $x = \frac{1}{2}$, then $1 + 1\frac{1}{2} + 2 + 2\frac{1}{2} + 3 + 3\frac{1}{2}$ *&c.* $- z = \infty^2$ = to the Square of Infinite.

And thus may you find the Ratio between the Sum of any aſcending arithmetical Progreſſion, and the Square of Infinite; and note that by Infinite, is always underſtood Infinite in General, or Infinite of the ſimpleſt Nature and loweſt Degree, $1+1+1+1.$ *&c.* unleſs ir be otherwiſe expreſſly declared.

Corol. 1. $\infty + \frac{1}{\sqrt{2x}}$ is the Root of a Square, which Square is equal to the Sum of any arithmetical Progreſſion aſcending in *infinitum.*

Examp. Let $x = 2$, then $\infty \times \frac{1}{2}$ (or an infinite number of $\frac{1}{2}$) is the Root of a Square equal to the Sum of that Progreſſion, *viz.* $1+3+5+7,$ *&c.* $= \overline{\tfrac{1}{2}+\tfrac{1}{2}+\tfrac{1}{2}+\tfrac{1}{2}\ \&c.}^{2}$.

Corol. 2. Let $a = \infty\, n$, $v = \infty\, m$. $t = \infty\, p$. Theſe values of a, v, t, being ſubſtituted in Lem. 2. will give you $2z = \infty p \times \overline{\infty\, n + \infty\, m} = \infty^{1} \times \overline{pn + pm}$, or $z = \frac{\infty^{2} \times \overline{pm + pn}}{2}$, and the common Difference $x = \frac{m - n}{p}$, as you may eaſily find from Lem. 1.

You may expreſs the Sum z otherwiſe, for $t = \frac{v - a}{x}$ by *Lem.* 1. Therefore $\frac{v - a}{x} \times \overline{v + a} = 2z$ by *Lemma* 2*d* that is $z = \frac{v^{2} - a^{2}}{2\,x} = \frac{\infty^{2} m^{2} - \infty^{2} nn}{2x} = \frac{\infty^{2} \times \overline{mm - nn}}{2z}$, *Corol.* 3. From *Prop.* 1. and *Corol.* 2. it is evident that in two arithmetical Progreſſions having the ſame common Difference x, the Sum of that whoſe Terms are infinite, is to the Sum of that whoſe Terms are finite, as $m^2 - n^2$ to 1. For the Sum of the firſt, by *Corol.* 2. is $\infty^2 \times \frac{m^2 - n^2}{2z}$, × the Sum of the other, by *Prop.* 1. $\infty^2 \times \frac{1}{2} x$.

Corol. 4. From hence may be ſolved this Problem. Any arithmetical Progreſſion being given as $a, a + x, a + 2x, a + 3x,$ *&c.* whoſe Terms are finite; to find another conſiſting of infinite Terms, that ſhall have the ſame Difference x, and whoſe Sum ſhall be equal to the Sum of the given Progreſſion. *Solution.* $m^2 - n^2 = 1$ from *Corol.* 3. *& ex hypotheſi.* Therefore $m^2 = 1 + n^2$; ſo to find m and n, is an indetermined Problem, whoſe Solution by the known Methods will give you $m = \frac{e^2 + 1}{e^2 - 1}$

$$n = \frac{2e}{e^2 - 1},$$

where e is any Number $(\triangle 1)$ taken at Pleaſure. So then theſe Values of m and n being ſubſtituted in the Values of a and v, of *Corol.* 2. will give you the firſt Term $a = \infty \times \frac{2e}{e^2 - 1}$, and the laſt $v = \infty \times \frac{e^2 + 1}{e^2 - 1}$, and calling the common Difference x, you will have a Progreſſion, each of whoſe Terms is infinite, and whoſe Sum ſhall be equal to the Sum of any other arithmetical Progreſſion, whoſe firſt Term is finite, and the common Difference x; and becauſe e is an arbitrary Number, therefore you have as many of ſuch Progreſſions as you pleaſe, to anſwer the *Problem.*

Examp. Let the given Progreſſion be $1 + 2 + 3 + 4 + 5$, *&c.* where $x = 1$, ſo the Sum thereof is $= \infty^2 \times \frac{1}{2}$ (as in *Ex.* 1. *Prop.* 1.) now taking $e = 2$, you will have $n = \frac{2e}{e^2 - 1} = \frac{4}{3}$, *&c.* $m = \frac{e^2 + 1}{e^2 - 1} = \frac{5}{3}$, which from the ſecond Expreſſion of *Corol.* 2. will give the Sum of the Series $(\infty \times \frac{4}{3},\ 1 + \infty\ \frac{4}{3},\ 2 + \infty\ \frac{4}{3},\ 3 + \infty\ \frac{4}{3}$, *&c.*) $\infty^2 \times \frac{m^2 - n}{2x} =, \infty^2 \times \frac{\frac{25}{9}\ \frac{16}{9}}{2x} = \infty^2 \times \frac{1}{2}$, which is the ſame with that of the given Progreſſion conſiſting of finite Terms.

Scho. The Problem may be made more general, and the Solution as eaſy, if it were required, that the Sum of the Series, conſiſting of infinite Terms, ſhould be to that of the other conſiſting of finite Terms in any given Ratio of r^2 to s^2; for then it muſt be to find m and n, ſuch that $m^2 - n^2 : 1 :: r^2 : s^2$.

Prop. 2. Let all Things be as in *Prop.* 1. except the laſt Term, which here we ſhall ſuppoſe

$v = \infty n$, in which Caſe I ſay $\left\{ z = \infty^2 \times \frac{nn}{2x} \right\}$

Examp. 1. Let $n = 2$, $x = 1$, $a = 1$, ſo the Progreſſion will be 1, 2, 3, 4, 5, *&c.* till the laſt Term be ∞ 2, I ſay the Sum $1 + 2 + 3 + 4 + 5$, *&c.* $= z = \infty^2 \times 2 =$ double the Square of Infinite.

§. II. In geometrical Progreſſion, let a denote the firſt, and v the laſt Term, the Ratio of the Terms, that of r to s, t the Number, and z the Sum of all the Terms of any geometrical Progreſſion, which will be a, $\frac{sa}{r}, \frac{s^2a}{r^2}, \frac{s^3a}{r^3}, \frac{s^4a}{r}.$

Of theſe five Things, *viz.* the firſt Term a, the laſt Term v, the Ratio of the Terms $\frac{r}{s}$, the number t, and the Sum of the Terms z, having any three, you may find the other two by theſe two *Lemma's*.

$$\left.\begin{array}{l} \textit{Lem. } 1.\ v. = \dfrac{\overline{s}|^{t-1}}{\overline{r}|^{t-1}} \times a \\ \textit{Lem. } 2.\ sz + ra = rz + v \end{array}\right\}$$

From *Lemma* 2 *d.* it follows that $z = \frac{ra - sv}{r - s}$ when the Progreſſion deſcends, that is, when $r < s$.

But when the Progreſſion aſcends, that is, when $r > s$, then from *Lemma* II. it will be $z = \frac{sv - ra}{s - r}$

Caſe 1. In deſcending Progreſſions, where the Number of Terms are infinite, the laſt Term v will be $= 0$.

Prop. 1. In all deſcending Progreſſions, whoſe Number of Terms are infinite, and firſt Term finite, the Sum of the Progreſſion is a finite quantity, *viz.* $\left. z.\ \frac{ra}{r-s} \right\}$

Corol. 1. Let $a = r$, then $z = \frac{r^2}{r-s}$

Examp. Let $r = 1$, $s = \frac{1}{2}$, then $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}$, *&c.* $= 2 = \frac{r^2}{r-s} = z$

Prob. Having the firſt Term a, and the Ratio of the Terms $\frac{r}{s}$ of any geometrical Progreſſion deſcending in *infinitum*, till $v = 0$, to find another Progreſſion deſcending in *infinitum*, whoſe firſt Term ſhall be any given Number *b*, and its Sum equal to the Sum of the given Progreſſion.

Sol. Let *m* to *n* be the Ratio of the Terms of the Progreſſion ſought; now $\frac{ra}{r-s}$ is the Sum of the given Progreſſion, by *Prop.* 1. and for the ſame reaſon $\frac{mb}{m-n}$ muſt be the Sum of the Progreſſion ſought; therefore from the Condition of the Problem ſought $\frac{mb}{m-n}=\frac{ra}{r-s}$ which gives $=\frac{m\times\overline{ra-rb+sb}}{ra}$, where *a*, *r*, *s*, *b*, are given, and *m* may be taken at Pleaſure; and ſo you have *n* from this Equation; and conſequently the Progreſſion ſought is found; for a Progreſſion is found, when you have the firſt Term *b*, and the Ratio of the Terms *m* to *n*.

Examp. Let $r=1$, $s=\frac{2}{2}$, $a=1$; $b=\frac{1}{2}$, I aſſume $m=\frac{2}{3}$ $(=b)$ ſo the Equation will give $n=\frac{4}{9}$, therefore $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}$, *&c.* $=\frac{2}{3}+\frac{4}{9}+\frac{8}{27}$, *&c.*

And here it is to be noted, that ſince *m* is taken at Pleaſure, the Problem is indetermined, and ſo you may find as many Progreſſions as you pleaſe, whoſe Sum ſhall be equal to the Sum of any given Progreſſion.

Schol. The Problem will be more general, if it be required that the Sum of the given Progreſſion $\frac{ra}{r-s}$ be to the Sum of the Progreſſion ſought $\frac{mb}{m-n}$ in any given Ratio h to k *viz.* $\frac{ra}{r-s} : \frac{mb}{m-n} :: h : k$, which gives $z = \frac{m \times ark + bhs - bhr}{akr}$; where m is at Pleaſure.

Caſe II. In aſcending Progreſſions, whoſe Number of Terms are infinite, the laſt Term v will be $= \infty \times e$; but firſt I ſhall conſider theſe Progreſſions in which $e = 1$, that is, where the laſt Term is an infinite Number of Units.

Prop. 2. $z = \infty \times \frac{s}{s-r} \left(= \frac{sv}{s-r}\right)$ for ra vaniſhes, becauſe a is finite, and v infinite.

But here it is carefully to be obſerved, that in all theſe Progreſſions it is neceſſary to make $a = r$, ſo that r will be the firſt, and s the ſecond Term of the Progreſſion: And indeed in all geometrical Progreſſions, there is a Convenience of expreſſing the Ratio of the Terms, by the two firſt Terms of the Progreſſion.

This being premiſed. it is evident that $\frac{s}{s-r}$ is the infinitely ſmall and equal Part, of which an infinite Number gives the Sum of any geometrical Progreſſion aſcending in *infinitum*.

Examp. 1. Let $r=1$, $s=3$, then $\frac{s}{s-r}=\frac{3}{2}$, which ſhews $z=\infty\times\frac{3}{2}$, that is $1+3+9+27+81$ &c. $=\frac{3}{2}+\frac{3}{2}+\frac{3}{1}+\frac{3}{2}$, &c.

Examp. 2. Let $r=1$, $s=\frac{3}{2}$, ſo $\frac{s}{s-r}=3$, Ergo $z=\infty\times 3$, that is, $1+\frac{9}{4}+\frac{27}{8}+\frac{81}{16}$ &c. $=3+3+3+3$, &c.

Problem. To find a Progreſſion, which ſhall have any given Number (a) for the equal Part of which its Sum conſiſts. *Solution*. Let r be the firſt, and s the ſecond Term of the Progreſſion ſought, then (becauſe $\frac{s}{s-r}$ is the equal Part for every Progreſſion) $\frac{s}{s-r}=a$ which gives $s=\frac{ra}{a-1}$, ſo taking the firſt Term r at Pleaſure, the ſecond ſhall be $\frac{ra}{a-1}$.

Examp. To find a Progreſſion, whoſe Sum ſhall be $=\infty\times 2$. In this Caſe $a=2$, ſo taking $r=1$, it will give $\frac{ra}{a-1}=2$ which ſhews that the Sum of a Progreſſion, whoſe firſt Term is 1, and ſecond Term is $2=\infty\ 2$, or $1+2+4+8$, *&c.* $=2+2+2+2$, *&c.*

Scholium. Becauſe r is taken at Pleaſure, therefore you may find as many Progreſſions as you pleaſe, whoſe Sums ſhall be all equal, becauſe each of them is $=\infty\times a$.

Prob. 2. Having any Progreſſion aſcending, to find another whoſe Sum ſhall be equal to the Sum of the given one. Let r be the firſt, s the ſecond Term of the given Progreſſion; and m the firſt, n the ſecond Term of the Progreſſion ſought, then $\infty\times\frac{s}{s-r}$ is the Sum of the given one, and $\infty\times\frac{m}{m-n}$ is the Sum of the Progreſſion ſought, *Ergo*, $\infty\times\frac{s}{s-r}=\infty\times\frac{m}{m-n}$, or $\frac{s}{s-r}=\frac{m}{m-n}$ which gives $n=\frac{rm}{s}$, m may be taken at Pleaſure, and ſo you have n.

Schol. So may you find a Progreſſion, whoſe Sum ſhall be to the Sum of the given one, in any given Ratio h to k, for then $\frac{m}{m-n} : \frac{s}{s-r} :: h : k$, ſo that $n = \frac{m \times \overline{sh \times rk - sk}}{sh}$

§. III. Before I proceed to other Progreſſions it will be neceſſary to ſubjoin ſome Things, which were omitted in § I. concerning arithmetical Progreſſion, and which ſhould have immediately preceeded § 2.

Lemma. $z = \infty^2 \times \frac{1}{2x}$, when $v = \infty \times 1$. by *Prop.* 1. of § 1.

Prob. Having the common Difference x, and the laſt Term $v = \infty \times 1$ of any arithmetical Progreſſion aſcending in *infinitum*; to find another arithmetical Progreſſion, whoſe Sum ſhall be equal to the Sum of the given one.

Let e be the common Difference, $\infty \times n$ the laſt, and y the Sum of all the Terms of the Progreſſion ſought; then $y = \infty^2 x \frac{nn}{2e}$: by *Prop.* 2. of § 1. Now becauſe $z = \infty^2 \times \frac{1}{2x}$ is the Sum of the given Progreſſion, therefore from the

the Condition of the Problem $\frac{1}{2x}=\frac{nn}{2e}$, ſo $e=x\,nn$; you may take n at Pleaſure, and ſo you have e, and conſequently the Progreſſion ſought.

Examp. 1. Let the given Progreſſion be 1, 2, 3, 4, 5, *&c.* to $\infty + 1$, where $x=1$, ſo $e=nn$; calling $n=2$, you have $e=4$ for the common Difference of the Progreſſion ſought, *viz.* 1, 5, 9, 13, *&c.* to $\infty \times 2$. I ſay then, that the Sums of theſe two Progreſſions are equal.

Corol. $n=\sqrt{\frac{e}{x}}$, ſo taking e at pleaſure, you have n, as in the former Example, if you call e, 2, then $n=\sqrt{2}$, ſo the Progreſſion ſought is, 1, 3, 5, 7, *&c.* to $\infty \times \sqrt{2}$, whoſe Sum ſhall be equal to $1+2+3+5+6$, *&c.* to $\infty \times 1$. So that in ſolving this Problem, you may either make the common Difference, or the infinitely ſmall Part (n) of the laſt Term, what you pleaſe; and becauſe one of the two may be taken at pleaſure, therefore the Problem is indetermined, and conſequently you may find as many Progreſſions as you pleaſe, whoſe Sums ſhall each of them be equal to the Sum of the given Progreſſion.

Scholium. In the ſame Manner, you may find as many Progreſſions as you pleaſe; the Sum of each of which ſhall be to the Sum of the given one in any given Ratio of k to h; for then

$n\,n$

$\frac{nn}{2e} : \frac{1}{2x} :: k : h$, which gives $ke = hxn^2$; ſo that of theſe two (e, n) taking one at pleaſure, you have the other from this Equation.

Problem. 2. Having the Sum $\infty^2 \times q$ of any Progreſſion, to find another that ſhall have a given common Difference e, and whoſe Sum ſhall be equal to the given Sum. Let $\infty \times n$ be the laſt Term of the Progreſſion ſought, then its Sum will be $= \infty^2 \times \frac{n^2}{2e}$; therefore by the Condition of the Problem $\frac{n^2}{2e} = q$. ſo that $n = \sqrt{2eq}$: but, e and q are given, therefore n is known.

Examp. Let $q = 1$, it is required to find a Progreſſion, whoſe Difference ſhall be 1, and its Sum $= \infty^2 \times 1$, or the Square of infinite; now becauſe $q = 1$, and $e = 1$, therefore $n = \sqrt{2}$, ſo that $\infty \times \sqrt{2}$ ſhall be the laſt Term of a Progreſſion 1, 2, 3, 4, 5, 6, 7, *&c.* whoſe Sum ſhall be $= \infty^2$; but by *Examp*. 2*d*. of *Prop*. 1. of § 1. if $x = \frac{1}{2}$, and $v = \infty$, then the Sum $z = \infty^2$; *Ergo* $1 + 2 + 3 + 4 + 5$, *&c.* to $\infty \times \sqrt{2} = 1 + 1\frac{1}{2} + 2 + 2\frac{1}{2}$, *&c.* to $\infty \times 1$.

Examp. 2. Let $q = \frac{1}{2}$, and $e = 2$, then $n = \sqrt{2}$, ſo that a Progreſſion, whoſe common Difference

ference is 2, and laſt Term $\infty\times\sqrt{2}$ ſhall have its Sum $=\infty^{2}\times=$ (by *ex.* 1. *pr.* 1.) $1+2+3+4+5$, *&c.* to ∞.

§. V. *Lemma* $\frac{1}{1-r|}e=1+er+\frac{e\times\overline{e+1}}{2}\times r^{2}+\frac{e\times\overline{e+1}\times\overline{e+2}}{2\times3}\times r^{3}+\frac{e\times\overline{e+1}\times\overline{e+2}\times\overline{e+3}}{2\times3\times4}\times r^{4}+\frac{e\times\overline{e+1}\times\overline{e+2}\times\overline{e+3}\times\overline{e+4}}{2\times3\times4\times5}\times r^{5}$, *&c.*

Let us now conſider the various Progreſſions that will ariſe out of this Series, and what relation they have to infinite in general or $\infty\times1$; and that I may proceed diſtinctly, I will reſolve it into ſeveral Caſes, beginning at the ſimpleſt, where $r=1$, and ſo proceed gradually to $r=2$, $r=3$, *&c.* And where the Series is neither in an arithmetical, nor yet in a geometrical Progreſſion, I ſhall endeavour to diſcover according to what increaſe the Progreſſion goes on; though, in general, that is plain enough from the *Lemma* of § 5.

CASE 1. Let $r=1$, which contains an infinite Number of other Caſes, according to the different Values of e, the Progreſſion whereof I ſhall ſhew in the following Articles.

Artic. 1. $(e=1.)\ \frac{1}{1-1}=1+1+1+1$, *&c.* $=\infty\times1$.

Artic. 2. $(e=2)\ \frac{1}{1-1|}{}^{2} = 1+2+3+4$, *&c.* $=\infty^{2}$

Artic. 3. $(e=3)\ \frac{1}{2-1|}{}^{3} = 1+3+6+10$, *&c.* $=\infty^{3}$

Artic. 4 $(e=4)\ \frac{1}{1-1|}{}^{4} = 1+4+10+20$, *&c.* $=\infty^{4}$.

Now you are to obſerve that the Terms of any one of theſe Progreſſions are made up of the Sum of the Terms of the Progreſſion next preceeding; for Inſtance, the third Term of the Progreſſion ∞^{2}, *viz.* 6, is the Sum of the three firſt Terms of the Progreſs $=\infty^{2}$; in like Manner the fourth Term (20) of Progreſſion in Article fourth, is the Sum of the four firſt Terms, in the Progreſſion of *Artic.* 3 *d.* and the 7*th.* Term of *Artic.* 3 *d.* is the Sum of the ſeven firſt Terms in *Artic.* 2 *d.* It is to be obſerved likewiſe, that the Terms of *Art.* 3*d.* are *triangular Numbers*, ſince they are the Sums of the natural Numbers of *Art.* 2*d.* and conſequently the Sum of the *triangular* Progreſſion, continued in *infinitum*, is equal to the *Cube* of *Infinite*. *Caſe* 2*d.* $r=2$.

Art. 1. $e = 1$, then $\frac{1}{1-2} = 1 + 2 + 4 + 8$, *&c.* $= \infty \times 2$.

Art. 2. $e = 2$, then $\frac{1}{1-2|}2 = 1 + 4 + 12 + 32$, *&c.* $= \infty^{2} \times 4$.

Art. 3. $e = 3$, then $\frac{1}{1-2|}3 = 1 + 6 + 24 + 80$, *&c.* $= \infty^{3} \times 8$.

Art. 4. $e = 4$, then $\frac{1}{1-2|}4 = 1 + 8 + 40 + 160$, *&c.* $= \infty^{4} \times 16$.

Note, That the Terms of *Art.* 1*ſt.* are in a geometrical Progreſſion in the Ratio of 1 to 2.

AND it is obſervable, that the Terms of any following *Article* are made by the Multiplication of the Terms of *Art.* 1*ſt.* into the reſpective Terms of that *Article* in *Caſe* 1. where *e* has the ſame Value. For Inſtance, the third Term of *Artic.* 2. is the Product of the third Term of *Artic.* 1. into the third Term of *Artic.* 2, of *Caſe* 1. and the fourth Term (*viz.* 160) is the Product of the fourth Term (*viz.* 8.) of *Artic* 2. into the 4*th* Term (*viz.* 20) of *Artic.* 4*th*, *Caſe* 1*ſt.* And univerſally, let *A* denote any Term in any *Artic.* of *Caſe* 1. *B* any Term in *Artic.* 1*ſt.* of *Caſe* 2 *d.* And *C* any Term in any of the following *Articles* of *Caſe*

Caſe 2d. I ſay $C = AB$, taking ABC in the ſame Order *i. e.* if C be the *5th* Term, then A and B muſt alſo be the *5th* Terms of their Progreſſions, and whatever Value (e) has in the Progreſſion, of which C is the Term, it muſt have the ſame in that Progreſſion where A is the Term. *Examp.* To find the *7th* Term of *Artic.* 3. of *Caſe* 2. Here C denotes the *7th* Term of a Progreſſion in which $e = 3$: This ſhews that A is the *7th* Term of *Artic.* 3. *Caſe* 1*ſt*, which multiplied into the *7th* Term B of *Artic.* 1, *Caſe 2d.* will give C the Term ſought.

Caſe 3d. $r = 3$

Art. 1. $e = 1$, then $\frac{1}{1-3} = 1 + 3 + 9 + 27$ &c. $= \infty \times \frac{1}{2}$.

Art. 2. $e = 2$, then $\left.\frac{1}{1-3}\right|2 = 1 + 6 + 27 + 108$, &c. $= \infty^{2} \times \frac{9}{4}$.

Art. 3. $e = 3$, then $\left.\frac{1}{1-3}\right|3 = 1 + 9 + 54 + 270$, &c, $= \infty^{3} \times \frac{27}{8}$.

Art. 4. $e = 4$, then $\left.\frac{1}{1-3}\right|4 = 1 + 12 + 90 + 540$, &c. $= \infty^{4} \times \frac{81}{16}$.

Note, The firſt Series is a geometrical Progreſſion going on in the Ratio of 1 to 3. And the following Progreſſions are made out of this firſt, with the reſpective ones of thoſe in *Caſe 1ſt* in all Reſpects, as thoſe of *Caſe 2d*, already explained: For let *A* be any Term in any *Artic.* of *Caſe* 1. *B* a Term of the ſame Order (with *A*) in *Artic.* 1. *Caſe* 3. and *C* a Term of the ſame Order in any of the *Articles* of *Caſe* 3*d*. I ſay, $C = AB$, where e has the ſame Value in *A* that it has in *C*, and ſo it is for all the ſucceeding Caſes *in infinitum*: The firſt Series of any Caſe, *r* (*viz.* where $e = 1$) is always a geometrical Progreſſion in the Ratio of 1 to r; the following Progreſſions are made by the Multiplication of the Terms of this firſt Series into the reſpective Terms of thoſe in *Caſe* 1. as has been ſhewn in *Caſe* 1 and 2*d*,

Scholium. To have the firſt Progreſſion of every *Caſe*, let $e = 1$, then $\frac{1}{1-r} = 1 + r + r^2 + r + r^4 + r^5$, *&c.*

But $1 + r + r^2 + r^3 + {}^4$, *&c.* $= \infty \times \frac{r}{r-1} = \frac{r}{r-1} + \frac{r}{r-1} + \frac{r}{r-1} + \frac{r}{r-1}$ *&c.*

BEFORE I conclude this Subject, it will be neceſſary to remove a very obvious and material Objection, *viz.* how it comes to paſs that the Sum of the natural Numbers $1 + 2$

$+3+4+5$, *&c.* ſhould be equal to $\frac{\infty^2}{2}$ or half the Square of Infinite, as it appears by *Examp.* 1. of *Prop.* 1. of § 1. And yet the ſame Sum is $= \infty^2$ or the whole Square of Infinite, as appears by *Art.* 2*d.* of Caſe 1. of the laſt *Lemma.* This ſeeming Contradiction may be reconciled, if we ſuppoſe $\infty \times 1$ to be the laſt Term in the Progreſſion of *Exam.* 1. *Prop.* 1. §. 1. which will make the Sum $\frac{\infty^2}{2}$; and $\infty \times \surd 2$ to be the laſt Term in the Progreſſion of *Art.* 2 *d.* of Caſe 1. of the laſt *Lemma*; for that will make the Sum of the Progreſſion $= \infty^2$, as appears by *Ex.* 1. of *Prop.* 2. § 3. So that though it be the ſame Progreſſion going on in *infinitum*, yet the one goes on to a greater Infinite, *viz.* $\infty \times \surd 2$, than the other, which ends at $\infty \times 1$.

But the plain way of reconciling the Matter depends on the common Rules of Multiplication. $1+2+3+4+5$, *&c.* $= \frac{1}{1-1}{}^{2} = \frac{1}{1-1} \times \frac{1}{1-1}$, but $\frac{1}{1-1} = 1+1+1+1$, *&c.* and therefore $1+2+3+4+5$, *&c.* $= \overline{1+1+1+1, \&c.} \times \overline{1+1+1+1+1, \&c.}$

Let us now make an actual Multiplication of ſix Terms only, thus

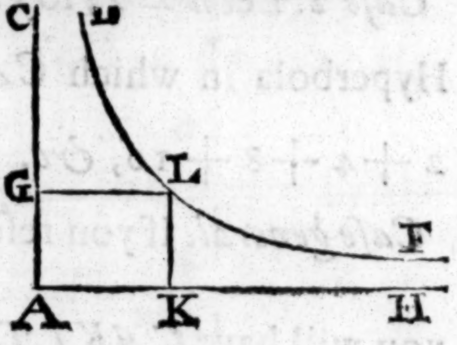
$1 +$

$$\begin{array}{l}
1+1+1+1+1+1\\
1+1+1+1+1+1\\
\hline
A.\,1+1+1+1+1+1\,B.\\
\qquad 1+1+1+1+1+1\\
\qquad\qquad 1+1+1+1+1+1\\
\qquad\qquad\qquad 1+1+1+1+1+1\\
\qquad\qquad\qquad\qquad 1+1+1+1+1+1\\
\qquad\qquad\qquad\qquad\quad C.\,1+1+1+1+1+1\,D.\\
\hline
1+2+3+4+5+6+5+4+3+2+1
\end{array}$$

FROM the Proceſs it is evident, that adding the ſeveral Colums, their Sums make 1, 2, 3, 4, 5, 6, which is the greateſt, *viz BC* (from which they deſcend in the ſame Order to *D*) and this *BC* is always the Number of Units in each Factor; ſo that if the Number of Units multiplied into it ſelf, had been 1000, then *BF*, the laſt Term of the Progreſſion, aſcending from *A* would have been 1000; and conſequently if the Number of Units multiplied into it ſelf, had been ∞ × 1, then *BC*, the laſt Term of the Progreſſion, would have been ∞ × 1. Now the Progreſſion deſcends from *BC* to *D*, in the ſame Order as it aſcends from *A* to *BC*; but *ABC* is the Sum of the Progreſſion 1, 2, 3, 4, 5, 6, *&c.* ending at *BC* = ∞ × 1; and this Sum *ABC* is but half the Product (of which *BCD* is the other) that is to ſay, half the Square of Infinite: Thus you ſee, that the Proceſs of *Art.* 2 of Caſe 1. agrees exactly with that of *Prop.* 1. of § 1. only that of *Art.* 2. Caſe 1. gives you the Progreſſion twice over, and ſo makes it double of what it is in *Ex.* 1. *Prop.* 1. § 1.

Schol. From this Solution it appears, that the Sum of the Progreſſion in *Art.* 3. Caſe 1. *viz.* $1+2+3+6+10+21$, *&c.* is not preciſely ∞^3 or the Cube of Infinite; for $\frac{1}{1-1}\times\frac{1}{1-1}\times\frac{1}{1-1}=1+\overline{1+1+1,\ \&c.}\times\overline{1+1+1+1,\ \&c.}\times\overline{1+1+1+1,\ \&c.}$ contains that Progreſſion oftner than once; and the ſame is to be conſidered in all the Progreſſions, except when $e=1$.

§ VI. The Arithmetick of Infinites applied to Quadratures of curvilinear Spaces.



LET *DLF* be any of the Hyperbola's, whoſe Aſymptots are *AC*, *AH*, let $AK=x$, $KL=y$, and the Equation comprehending all the Hyperbola's $y x^n = 1$.

By

By the common Methods, And alſo $\left\{ \begin{array}{l} CAKLD = \frac{1}{1-n} \times x^{1-n} \\ HAGLF = \frac{n}{n-1} \times x^{1-n} \end{array} \right\}$

Corol. 1. Let $x = 1$, then, $CAKLD = \frac{1}{1-n}$, and $HAGLF = \frac{n}{n-1}$ }

Caſe 1. Let $n = 1$, ſo the Equation will be $yx = 1$ for the common Hyperbola, in which $CAKLD. = \frac{1}{1-1} = 1 + 1 + 1$, *&c.* $= \infty \times 1$. And likewiſe $HAGLF = \frac{1}{1-1} = 1 + 1 + 1 + 1$, *&c.* $= \infty \times 1$. From whence it appears, that the Area of the *Apollonian* Hyperbola is infinite both ways.

Caſe 2. Let $n = 2$; ſo $yx^2 = 1$ defines the next Hyperbola in which $CAKLD = \frac{1}{1-2} = 1 + 2 + 4 + 8 + 16$, *&c.* $= \infty \times 2$.

Caſe general. If you reſolve $\frac{1}{1-n}$ into a Series, you will have $CAKLD = \frac{1}{1-n} = 1 + n \times n^2 + n^3 + n^4 + n^5$, *&c.* and becauſe the Terms of this Series are in a geometrical Progreſſion aſcending (ſuppoſing $n < 1$) in the Ratio of 1 to n, there-

n, therefore by *Prop.* 2. §. 2. the Sum thereof muſt be $1 + n + n^2 + n^3 + n4 + n^5$, &c. $= \infty \times \frac{n}{n-1}$

Now becauſe $n < 1$, therefore $\frac{n}{n-1}$ is < 1; therefore in all theſe Hyperbola's (in which $n < 1$) the Area *CAKLD* will be an infinite Number of equal Parts, each of which (*viz.* $\frac{n}{n-1}$ is greater than 1. And hence is underſtood the Meaning of the Geometers, who call theſe Spaces greater than Infinite, that is, greater than $\infty \times 1$ or Infinite in general.

Corol. 2. In all the Hyperbola's (except the *Apollonian*) the infinite Area *FLKLD* (adjacent to the Aſymptote *AC*) is equal to an infinite Number of the finite Area *HAGLF* (adjacent to the Aſymptote *AH*) in the ſame Hyperbola.

Demonſtration. $CAKLD = \infty \times \frac{n}{n-1}$ by the general Caſe, but $HAGLF = \frac{n}{n-1}$ by *Corol.* 1. § 6. Therefore

$$\left\{ CALD = \infty \times HAGLF \right\} \quad Q.\ E.\ D.$$

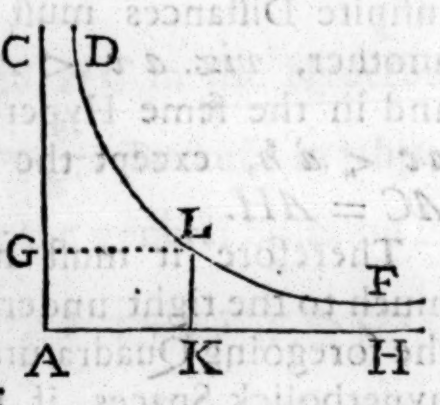
Problem. Let $\infty \times e$ be the infinite Area *CAKLD* of any given Hyperbola, it is required to find another Hyperbola, whoſe infinite

Space ſhall be to the Space ($\infty \times e$) of the given Hyperbola in any given Ratio, as of p to 1. *Sol.* Let $y\,x^n = 1$ be the Equation of the Hyperbola ſought; then by the general Caſe foregoing it's Area is $= \infty \times \frac{n}{n-1}$; therefore by the Condition of the Problem, $\frac{n}{n-1} : e :: p : 1$; which will give $n = \frac{pe}{pe-1}$; ſo that $y\,\overline{x}|^{\frac{pe}{pe-1}} = 1$ is the Equation to the Hyperbola ſought; but p and e are given Numbers, and therefore this is a known Equation, and conſequently the Hyperbola defined by it is alſo known.

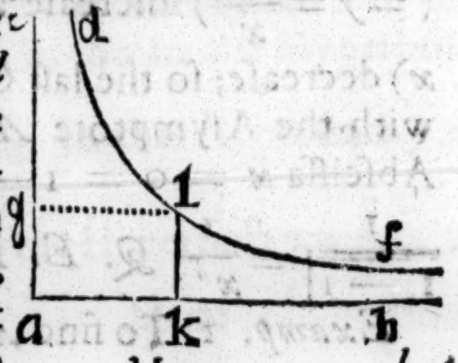
Examp. To find an Hyperbola, whoſe Area ſhall be to that of the Apollonian, as 3 to 1. Now the Apollonian is $= \infty \times 1$, ſo $e = 1$, and $p = 3$, *Ergo* the Equation is $y\,x^{\frac{3}{2}} = 1$, whoſe Area by the general Caſe foregoing is $= \infty \times 3$, which is triple of the Apollonian.

Prob. 2. To find an Hyperbola, whoſe interminated Space *HKLF* ſhall be equal to any given Number (a) let $y\,x^n = 1$ define the Hyperbola ſought, where $n < 1$, then $HAGLF = \frac{n}{n-1}$ putting $y = 1$ $(= GL = GA)$ by *Corol.* of § 6. and becauſe $AGLK = 1$, therefore it will be $HAGLF - AGLK$ $(= HKLE) = \frac{n}{n-1} - 1$. Therefore by the Condition of the

Problem $\frac{n}{n-1} - 1 = a$, which gives $n = \frac{a+1}{a}$; ſo the Equation to the Hyperbola ſought, is $\overline{y\,x}\Big|^{\frac{a \times 1}{a}} = 1$.



Let (as before) *AC*, *AH* be the Aſymtotes of any Hyperbola *DLF* defined by this Equation $y\,x^n = 1$, in which the Abſciſſa $AK = x$, and Ordinate $KL = y$, and n is ſuppoſed either equal to, or greater than Unity. 1°. It appears that in all Hyperbola's the interminate Space *CAKLD* is infinite, and the interminate Space *HAGLF* (except in the *Apollonian* where $n = 1$) is finite. 2°. In every Hyperbola, one Part of it continually approaches nearer and nearer to the Aſymptote *AC*, and the other part continually nearer to the other Aſymptote *AH*; that is, *LD* meets with *AC* at a Point infinitely diſtant from *A*, and *LF* meets with *AH* at a Point infinitely diſtant from *A*.



3°. In two different Hyperbola's *DLF*, *dlf*, if we ſuppoſe n to be greater in the Equation of *dlf*, than it is in the Equation of *DLF*, then *LD* ſhall meet ſooner with *AC* than *l d* with *a c*, but *L F* ſhall be longer in meeting with *A H* than *l f* with *a h*. Therefore ſince theſe Meetings are at infinite Diſtance from *A* and *a*, it follows neceſſarily that theſe infinite Diſtances muſt be one greater than another, *viz.* $ac < AC$, and $ah > AH$; and in the ſame Hyperbola $AC < AH$ and $ac < ah$, except the *Apollonian*, in which $AC = AH$.

Therefore it muſt needs contribute very much to the right underſtanding the Scope of the foregoing Quadratures of theſe interminate hyperbolick Spaces, if we can determine the Length between the Centre *A* or *a*, and the Point of Concourſe of either Part of the Hyperbola, with that Aſymptote to which it approaches.

Problem 3. To find the Point *C* where *L D* meets with the Aſymptote *AC*, and the Point *H* where *L F* meets with the Aſymptote *A H*, for any Hyperbola *D L F*, whoſe Equation $y x^n = 1$ is given.

Solution. It is evident, that the Ordinates *L K* $(= y = \frac{1}{x^n})$ increaſe as the Abſciſſa's *AK* $(= x)$ decreaſe; ſo the laſt Ordinate muſt co-incide with the Aſymptote *AC*, in which Caſe the Abſciſſa $x = 0 = 1 - 1$. Therefore $AC = \frac{1}{\overline{1-1}|_n} = \frac{1}{x^n}$ *Q. E. I.*

Examp. 1. To find *AC* in the common *Hy-*

perbola $y=\frac{1}{x}$. Becauſe in this $n=1$, therefore $NC=\frac{1}{1-1}=$ (by *Art.* 1. of § 5.) $1+1+1+1$, *&c.* $=\infty\times 1$.

Examp. 2. To find *AC* in the Hyperbola whoſe Equation is $y=\frac{1}{x^2}$. Becauſe in this $n=2$, therefore from the general Solution $AC=\frac{1}{\overline{1-1}|^{2}}=1+2+3+4+5+6$, *&c.* $=\frac{\infty^2}{2}$; That is, *AC* is an infinite Number of equal Parts, each of which is $\infty\,\frac{1}{2}$: So that *AC* in this is an infinite Number of equal Parts, each of which is $\frac{1}{2}$ *AC* in the common Hyperbola.

Schol. If we could give the preciſe Sum of the Series in *Art.* 3, 4, *&c.* of *Caſe* 1. of § 5, we ſhould then have *AC* for all the other Hyperbola's; but that is not eaſily to be done, as is declared in the Scholium at the End of § 5. Only this much we ſee (by *Caſe* 4. of § 5.) That *AC* increaſes as the Powers of ∞ whoſe Exponents are n. So in the Hyperbola $y=\frac{1}{x}$, *AC* is as ∞; in the Hyperbola $y=\frac{1}{x^2}$, *AC* is as ∞^2 in the Hyperbola $y=\frac{1}{x^3}$, *AC* is ∞^3; and ſo on.

Part. 2*d.* To find AH when LF meets with it's Aſymptote AH. Here we muſt conſider GL ($=x$) as the Ordinate, and AG ($=y$) as the Abſciſſa. Now when GL becomes AH, then $y=0$. But univerſally GL ($=x$) $=\frac{1}{y|^{\frac{1}{2}}}$ therefore $AH=\frac{1}{0\,|^{\frac{1}{n}}}=\frac{1}{\overline{1-1}|^{\frac{1}{n}}}$; put $\frac{1}{n}=e$, and then by *Lem.* of § 5. $AH=$

$$\frac{1}{1-1}e=1+e+\frac{e\times\overline{e+1}}{2}+\frac{e\times\overline{e+1}\times\overline{e+2}}{2\times3}+\frac{e\times\overline{e+1}\times\overline{e+2}\times\overline{e+3}}{2\times3\times4}, \text{ \&c.}$$

But ſince we cannot aſſign the Sum of theſe Progreſſions, therefore this Series is of no Uſe in the Solution of the Problem. We ſhall therefore conſider the Problem (as to the finding AH) under two Caſes, firſt when n is an Integer, and ſecondly when it is a Fraction.

Caſe 1. When n is an Integer. So if $n=1$ (as in the common Hyperbola) then AH ($=\frac{1}{1-1}=1+1+1$, &c. $=\infty\times1=AC$. If $n=2$, then AH ($=\frac{1}{\overline{1-1}|^{\frac{1}{2}}}=\sqrt{\frac{1}{1-1}}=\sqrt{\infty\times1}$) is a mean Proportional between 1 and infinite. If $n=3$, then AH ($=\frac{1}{\overline{1-1}|^{\frac{1}{3}}}=\sqrt[3]{\frac{1}{1-1}}=\sqrt[3]{\infty\times1}$

is the firſt of two mean Proportionals between 1 and infinite. And univerſally if between 1 and Infinite there be ſuppoſed as many mean Proportionals as there are Units in $n - 1$ then $AH (= \frac{1}{\overline{1-1}}\Big|\frac{1}{n} = \sqrt[n]{\frac{1}{1-1}} = \sqrt[n]{\infty \times 1})$ ſhall be the firſt of theſe Means.

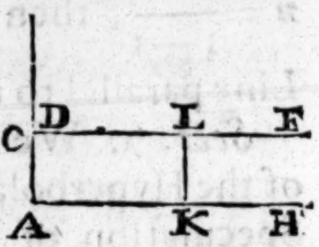
Caſe 2d. When n is a Fraction, ſuppoſe $n = \frac{p}{q}$, but $p < q$, becauſe we always ſuppoſe $n < 1$ except in the *Apollonian* where $n = 1$. So then $\frac{1}{n} = \frac{q}{p}$, therefore $AH = \frac{1}{\overline{1-1}}\Big|\frac{p}{q} = \sqrt[p]{\frac{1}{\overline{1-1q}}} = \overline{\infty \times 1}\Big|\frac{q}{p} = \sqrt[p]{\infty \times 1}\,|^{q}$. So for *Ex.* if $n = \frac{3}{2}$, *viz.* $p = 3$, $q = 2$, then $AH = \sqrt[3]{\overline{\infty \times 1}\,|^{2.}}$

Corol. From both theſe Caſes, it is evident, that the greater Number we ſuppoſe n to be, ſo much the ſooner will LF meet with AH: for the greater we ſuppoſe n to be ſo much the leſs will $\frac{1}{n}$ or $(\frac{q}{p})$ be; but $\frac{1}{n}$ (or $\frac{q}{p}$) is the Exponent of $\frac{1}{1-1}$ (or ∞) which gives the Value of AH. *Ergo* the greater that n is, the ſhorter will AH be. *Q. E. D.*

Schol. Since in all Hyperbola's $AH=\frac{1}{1-1}\,{}^{n}$ $=\frac{1}{1-1|^{\frac{q}{p}}}=\sqrt[p]{\frac{1}{1-1}}\Big|^{q}=\sqrt[p]{\infty\times1}\Big|^{q}$ is ſome mean Proportional between 1 and $\infty\times1$, it is worth the while to conſider what ſort of Numbers theſe Means are, *viz.* whether they be finite, infinite, or neither. It is certain they cannot be finite; for then it would follow that the Square of a finite Number ſhould be infinite, which is abſurd. Nor can they be properly eſteemed infinite, for the equal part of ſuch an Infinite muſt be 1 divided by ſuch a Mean.

Demon. Let a denote that equal part, then $\infty\times a=\sqrt[p]{\infty^{q}\,1}$, which gives $a=\frac{1}{\sqrt[p]{\infty}\,\big|^{p-q}}$ *Q. E. D.* (*Ex. gra.*) $\sqrt{\infty\times1}$ or a Mean between 1 and infinite muſt be $\infty\times\frac{1}{\sqrt{\infty}}$ or an infinite Number of equal Parts, each of which is $\frac{1}{\sqrt{\infty}}$ or 1 divided by the Mean between 1 and $\infty\times1$. So that this equal part $\frac{1}{\sqrt{\infty\times1}}$ is neither finite, nor infinitely ſmall, nor infinitely great, and conſequently the mean $\sqrt{\infty\times1}$ is not properly an infinite Number. And hence we ſee that there are Progreſſions of Numbers whoſe Sums are neither finite nor infinite, but between both. *Ex. gr.* $\sqrt{\infty \times 1}$ where $p = 2, q = 1$, ſo $\sqrt{\infty \times 1} \frac{8}{\overline{1-1}|^{\frac{1}{2}}} = 1\frac{1}{2} + \frac{1}{2} + \frac{3}{8}\,\frac{5}{16} + \frac{3}{1}\,\frac{5}{28}$ &c. is a Number neither finite nor infinite: And ſo all the Means between 1 and Infinite, are neither finite nor infinite.

I have hitherto conſidered only theſe Caſes in which $n =$ or < 1; let us now ſee what would be the Reſult if we ſuppoſe $n = o$. It is evident then that the general Equation, *viz.* $yx^n = 1$ to all Hyperbola's would, in this Caſe be $y = 1$, ſo that the Hyperbola's will in this Caſe degenerate into a ſtraight Line *DLF*, parallel to *AH*. But conſidering it as the ſimpleſt Hyperbola, let us conſider at what diſtance it will meet with its Aſymptotes. Now univerſally $AC = \frac{1}{x|n}$ by the *Sol.* of part. 1. of *Prob.* 3.) when $x = 0$, but $n = 0$ by Suppoſition: *Ergo*, be x what it will, $x^0 = 1$, *Ergo*, $AC = 1$, 2°. From the Solution of part 2. of *Prob.* 3. we have found that



$$AH = \frac{1}{1-1}\Big|^{\frac{r}{n}}$$ which in this Caſe will give

$$AH = \frac{1}{1-1}\Big|^{\frac{1}{0}} = \frac{1}{1-1_{\overline{1-1}}},$$ that is $AH = \infty|^{\infty}$ or the infinite Power of Infinite.

Corol. 1. Conſidering a ſtraight Line as an Hyperbola, it can have but one Aſymptote, *viz. AH,* whoſe Concurſe with *A H* is at greater Diſtance from *A* than any other Hyperbola whatſoever.

Corol. 2. All the Hyperbola's $y\,x^n = 1$ whoſe Aſymptotes are *AC, AH* muſt interſect *DF,* and the greater *n* is, ſo much the more they bend towards *AH* and receed from *DF*; and as they all interſect *DF,* ſo they all interſect one another in one Point only; and after the Interſection, that, in which *n* is greateſt, ſtill falls loweſt or approaches neareſt to *AH.*

Schol. If you ſuppoſe *n* infinitely great, *viz.* $n = \frac{1}{1-1}$, then the Hyperbola will be a ſtraight Line parallel to the Aſymptote *AC.*

Sect. 7. We have now finiſhed this Buſineſs of the Hyperbola's which has afforded us a new Speculation of Numbers, *viz.* of ſuch as are neither finite nor infinite, which deſerves to be conſidered better than either my Time or my Capacity will permit. However, I ſhall here ſet down a few Thoughts about them, till I have more Leiſure to proſecute them.

Firſt then, to diſtinguiſh them from finite and infinite, I ſhall call them *indefinite* Numbers, and denote them by this Sign ∝.

2°. *Indefinite* Numbers I ſuppoſe to be intermediate Numbers, lying between finite and infinite: For as we do not deſcend from 1 to 0 at one Step, but muſt paſs through an infinite Series of Fractions, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, *&c.* So it is impoſſible that in aſcending from 1 to ∞, we ſhould paſs immediately from finite to infinite; therefore the Steps between theſe two are indefinite Numbers; thus before we arrive from 1 to $\infty\ 1$, we muſt come at $\surd\ \infty\ 1$, and before we come at $\sqrt{\infty\ 1}$ we muſt firſt come at $\sqrt[3]{\infty \times 1}$ and ſo on.

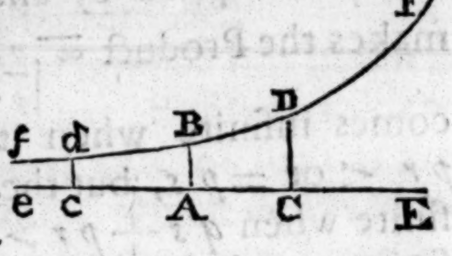
3°. The Rules for the Arithmetick of Indefinites may be made after the ſame manner as is done for that of Infinites by D. *Chyne.* To which I ſhall add, that an Indefinite as $\infty\,\frac{q}{p}$ or $\overset{p\qquad q}{\overline{\surd\ \infty\,|}}$ multiplied by another Indefinite as $\overline{\infty}|\frac{r}{s}$ makes the Product $\overline{\infty}\big|\frac{q}{p}+\frac{r}{s}$ this Product becomes infinite when it happens that $q\,s + p\,r <$ or $= p\,s$, but the Product is only indefinite when $q\,s + p\,r > p\,s$. And if an indefinite as $\overline{\infty}|\frac{q}{p}$ be divided by another indefinite as $\overline{\infty}|\frac{r}{s}$, the Quotient $\overline{\infty}|\frac{p}{q}-\frac{r}{s}$ is infinite when $q\,s - p\,r$ is $<$ or $= p\,s$; but it is finite when $q\,s - p\,r = 0$, and indefinite when $q\,s - p\,r > p\,s$.

Note, THAT in expreſſing an indefinite Number by $\overline{\infty}|\frac{p}{q}$, I always ſuppoſe the

Numerator q leſs than the Denominator p; for if q be either $<$ or $= p$, then $\overline{\infty}|\frac{p}{q}$ is an Infinite. That theſe Numbers are not infinite, may be thus demonſtrated. If (*ex. gr.*) $\sqrt{\infty \times 1}$ were infinite, then $\infty \times \frac{1}{\sqrt{\infty}}$ $(= \sqrt{\infty \times 1})$ muſt be infinite, and alſo $\frac{1}{\sqrt{\infty}}$ muſt be $= 0$; *Ergo* $\infty \times 0 = \infty \times \frac{1}{\sqrt{\infty}}$ $(= \sqrt{\infty}) =$ infinite, but $\infty \times 0 = 1$; *Ergo* $1 =$ infinite, which is abſurd.

Sect. 8. Containing ſome miſcellaneous Things relating to Infinites.

Lem. Let *FBD d e* be a Logarithmick Curve, whoſe firſt Ordinate $AB = 1$, Abſciſſa $AC = x$, Ordinate $CD = y$, and Aſymptote *E e*: Now from the known Property of this Curve, it follows,



1°. THAT *AC* are the Logarithms of *CD*, *i.e.* if you make the Ordinate $y = CD$ repreſent a Number, then its Abſciſſa x $(= AC)$ ſhall be the Logarithm thereof.

2°. THAT the Logarithm of 1 is nothing; for the firſt Ordinate $AB = 1$, but its Abſciſſa is $= 0$, therefore (by 1°) the Log. of $1 = 0$.

3°. THAT the Ordinates cd (to the left of AB) denoting Fractions, their reſpective Abſciſſa's Ac are the Logarithms of theſe Fractions: So that as x is the Log. of any Integer, CD, in the like manner $-x$ denotes the Log. of any Fraction (cd).

4°. THAT the Log. of any Fraction $\frac{1}{n}$ is equal to —Log. of n. So that the Log. of any Fraction (whoſe Numerator is 1) is equal in Magnitude to the Log. of that whole Number, which is the Denominator; there being no Difference between the Log. of that Fraction $\frac{1}{n}$ and the Log. of this Integer n, but that this is $+x$ (becauſe it lies to the right from A towards E) and the other $-x$ becauſe it lies to the left from A towards e.

Demonſtration. $\frac{1}{n}$ ſignifies 1 divided by n, therefore by the Rules of Diviſion $l:1 - l:n =$ Log. of the Quotient, *viz.* $l:\frac{1}{n}$. But $l:1 = 0$ (by 2°) *Ergo* $-l:n = l:\frac{1}{n}$. *Q.E.D.*

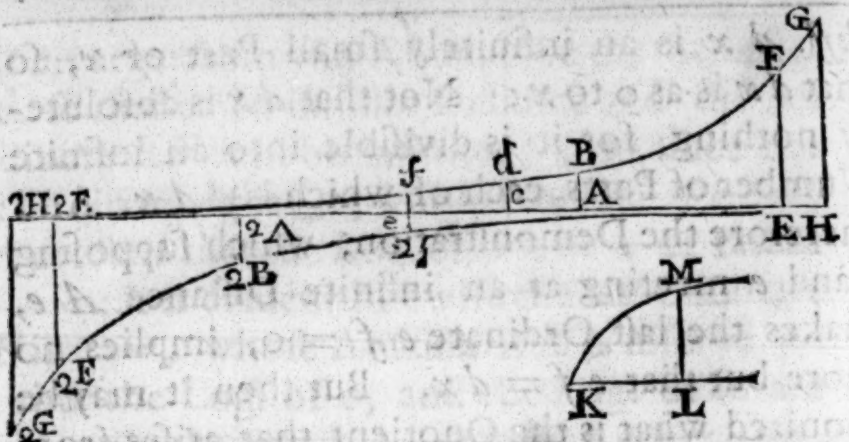
5°. If this Curve be continued (*utrinque*) from B *in infinitum*, then f will meet with the Aſymptote at an infinite Diſtance Ae: But it will diverge from the Aſymptote on the other ſide, ſo that at an infinite Diſtance AE the laſt Ordinate EF will be infinite. And ſince the laſt Ordinate ef is $=0$, whoſe Abſciſſa $Ae=\infty$, it is evident that the Log. of o is $=\infty$ or rather $=-\infty$: It is evident likewiſe the laſt Ordinate EF is $=\infty$, whoſe Abſciſſa AE is alſo $=\infty$. So that the Log. of o, and the Log. of ∞ are equal, only the one is $+\infty$ and the other $-\infty$.

Prop. 1. $\frac{1}{0}=\infty=1+1+1+1+1$, *&c.* This may ſeem abſurd, but the Demonſtration is evident from the foregoing *Lemma*. For $\frac{1}{1-1}$ $=\frac{1}{0}$. Let then $\frac{1}{1-1}=y$. That is $\overline{1-1}|^{1}=y$. This reduced to a logarithmical Equation gives $-1\times l:\overline{1-1}=l:y$; that is $-1\times l:0=l:y$. But $l:0=-\infty$ (by *Art.* 5. of the *Lem.*) *Ergo* $-1\times-\infty\ l:y$, that is $+\infty=l:y$. Therefore y is infinite ($ſc.\ y=\infty$) for no Number has an affirmative and infinite Logarithm, except an infinite Number. Since then $y=\infty$, then $\frac{1}{1-1}=\infty$ that is $\frac{1}{0}=\infty$. *Q. E. D.*

Schol. But by o cannot be underſtood abſolute nothing, for an infinite Number of abſolute Nothings cannot make 1; but by o is underſtood an infinitely ſmall Part, as in the *calc. diff.*

diff. dx is an infinitely ſmall Part of x, ſo that dx is as 0 to x: Not that dx is abſolutely nothing, for it is diviſible into an infinite Number of Parts, each of which is ddx. And therefore the Demonſtration, which ſuppoſing f and e meeting at an infinite Diſtance Ae, makes the laſt Ordinate $ef = 0$, implies no more but that $ef = dx$. But then it may be inquired what is the Quotient that ariſes from the Diviſion of 1 by abſolute Nothing. I ſay, there is no Quotient becauſe there is no Diviſion: Therefore it is a Miſtake to ſay the Quotient is 1 or Unity undivided, which is demonſtrably falſe, neither is the Quotient $= 0$. For properly ſpeaking there is no Quotient, and therefore it is an Error to aſſign any. In like manner, it is an Error to ſay, that $0 \times a$ makes the Product 0; for properly ſpeaking there is no Product. It is true, this of Multiplication has no influence upon Practice, but that of Diviſion has.

FROM hence it appears, that a Curve is ſaid to meet with its Aſymptote, when the Ordinate is infinitely little.



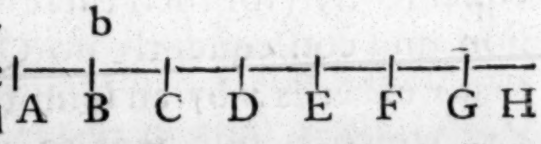
THIS ſame Notion does explain how it comes to paſs, that 1 divided by a negative Number gives a Quotient greater than infinite. For if (when you call $LM = x$, $KL = y$.) $fe = dx$, then becauſe the Points f and e are infinitely near, we may conceive the Logarithmick Curve continued as interſecting AH in the Point e; ſo that $FBf\,2f\,2B\,2F\,2G$ makes but one continued Curve, whereof the Part above repreſents the affirmative Numbers by its Ordinates, and the negative Numbers are repreſented by the Ordinates of the Part below; but A is the Beginning of the Axis for the Logarithms of both, *viz.* AE, AH, *&c.* are the Logarithms of the affirmative Integers FE, GH, *&c.* Ac the Logarithm of any affirmative Fraction, Ae the Logarithm of an infinitely little Fraction, $fe = dx$. $A\,2A$, $A\,2E$, *&c.* the Logarithms of the negative Numbers, $2A\,2B$, $2E\,2F$, *&c.*

Now let n be any Number greater than Unity, then $1 - n$ will be a negative Number; let us now ſee what the Quotient of $\frac{1}{1-n}$ will be, let it be y, ſo $\frac{1}{1-n} = y$, *i. e.* $\overline{1-n}|^{-1} = y$, this reduced to a logarithmick Equation, the Equation gives $-1 \times l : 1 - n = l : y$.

But $1 - n$ is a negative Number, ſuppoſe (*ex. gr.*) of $2A2B$, then $A2A$ is $= l : -n$, therefore $-1 \times A2A = l.y.$ but $A2A < Ae$, and $Ae = -\infty$, therefore $A2A < -\infty$. Let then $A2A = -\infty \times a$ (where $a < 1$) then $-1 \times -\infty a = l : y$, that is $\infty \times a = l : y$, but $\infty \times a < \infty$, *Ergo* y is a Number greater than infinite.

And here it is obſervable, that there are affirmative Numbers leſs than nothing, denoted by the ſeveral Powers of dx, as dx^2, dx^3, *&c.* or by the ſecond, third, *&c.* Differences, and theſe Numbers may be aptly repreſented by the Ordinates of the logarithmick Curve, continued from f towards $2H$ when dx^n is affirmative, or from $2f$ towards H when dx^n is negative.

Another way of explaining what is meant by $\frac{1}{0} = \infty$.



Let AH produced indefinitely be divided into equal Parts AB, BC, CD, DE, *&c.* ſo that a Part of this Line ſhall denote any Number, ſuppoſing $AB=1$, let then x denote any Number, (*ex. gr.*) Let $x=AB$, and $y=Ab$, ſo $\frac{1}{y-x}=\frac{1}{y}+\frac{x}{y^2}+\frac{x^2}{y^3}+\frac{x^3}{y^4}+\frac{x^4}{y^5}$, *&c.*

Now ſuppoſe b infinitely near to B, then $y-x=Bb=dx$, ſo that $\frac{1}{y-x}=\frac{1}{dx}$. But $x=dx+dx+dx$, *&c.* That is $1\times x=dx\times 1+1+1+1$, *&c.* And therefore $\frac{1}{dx}=\frac{1}{x}+\frac{1}{x}+\frac{1}{x}+\frac{1}{x}$ *&c.* But by Suppoſition $x=AB=1$. *Ergo* $\frac{1}{dx}=1+1+1+1$, *&c.* $=\infty\times 1$. But $\frac{1}{0}=1+1+1+1$, *&c.* *Ergo* $\frac{1}{0}=\frac{1}{dx}$, or $dx=0$, but dx is not abſolute Nothing, and therefore when we ſay $\frac{1}{0}=1+1+1$, *&c.* 0 does not denote abſolute Nothing, but only dx, or an infinitely ſmall Part of x. And therefore when (in the Quotient $\frac{1}{y-x}$) we ſay let $y=x$, the Meaning is not that y is abſolutely equal to x, (for then there would be no Diviſion, and conſequently no Quotient) but only that y exceeds x by an infinitely ſmall Quantity dx, which is ſufficient to make them equal.

I have hitherto conſidered no Progreſſions of Numbers except theſe that are in arithmetical and geometrical Progreſſions, and theſe that ariſe from $\frac{1}{1-1}\Big| e$ as in § 5. I ſhall now proceed to treat of other Progreſſions, and theſe, which offer themſelves firſt, are the ſeveral Powers of Numbers in an arithmetical Progreſſion. Let then z denote the Sum of any arithmetical Progreſſion $a, a+x, a+2x, a+3x, a+4x,$ *&c.* A the Sum of their Squares, B the Sum of their Cubes, C the Sum of their Biquadrates, *&c.* that is, let

$$z = a + (a+x) + (a+2x) + (a+3x) + (a+4x) \; \&c.$$

$$A = a^2 + \overline{a+x}|^2 + \overline{a+2x}|^2 + \overline{a+3x}|^2 + \overline{a+4x}|^2 \; \&c,$$

$$B = a^3 + \overline{a+x}|^3 + \overline{a+2x}|^3 + \overline{a+3x}|^3 + \overline{a+4x}|^3 \; \&c.$$

$$C = a^4 + \overline{a+x}|^4 + \overline{a+2x}|^4 + \overline{a+3x}|^4 + \overline{a+4x}|^4 \; \&c.$$

And ſo on to higher Powers. Now in order to find the Sum of any of theſe Progreſſions aſcending till the laſt Term is $= \infty \times 1$, I ſhall premiſe theſe following *Lemma's*, by the Help whereof you may find z, A, B, C, *&c.* for any Number of Terms. Calling therefore t the Number of Terms in each Series, and v the laſt Term of the firſt, whoſe Sum is z. I ſay

Lem. 1. $z = \frac{v^2 \times xv + ax - a2}{2}$

$$\textit{Lem. } 2.\ A = \frac{\overline{v+x}|^3 - a^3 - tx^3 - 3x^2 z}{3x}$$

$$\textit{Lem. } 3\ B. = \frac{\overline{v+x}|^4 - a^4 - tx^4 - 4x^3 z - 6x^2 A}{4x}$$

$$\textit{Lem. } 4.\ C = \frac{\overline{v+x}|^5 - a^5 - tx^5 - 5x^4 z - 10x^3 A - 10x^2 B}{5x}$$

In each of which $tx = v + x - a$ (by *Lem.* 1. § 1.) and it is eaſy to continue theſe *Lemma's* for higher Powers.

Before we apply theſe *Lemma's* to the finding the Sum of any Progreſſion whoſe laſt Term is infinite, it will be neceſſary to ſubſtitute the Values of t, z, A, B, C, *&c.* When you have made this Subſtitution, you muſt reject out of the *Lemma's* every Term in which v does not occur; for the laſt Term being infinite will make v an indefinite Number; and ſince all the other Terms (in which v is not) are finite, therefore they are as nothing in Reſpect of v, and conſequently to be rejected. So, for Progreſſions whoſe laſt Term is $= \infty$, the *Lemma's* will be

$$\textit{Lem. } 1.\ z = \frac{v^2 + xv}{2x}$$

$$\textit{Lem. } 2.\ A = \frac{v^3 - x^2 v}{3x} - \frac{v^2 - xv}{2}$$

Lem. 3. $B = \frac{v^4 - 2xv^3 + x^2v^2 + 2x^3v}{4x}$

Lem. 4. $C. = \frac{v^5}{5x} + \frac{15v^4 + 10xv^3 - 59x^3v}{30}$

LET us now conſider what the Value of v is, in each of the *Lemma's A, B, C, &c.* An Example or two will make the Thing plain.

Ex. 1. Let ⎧ 1, 2, 3, 4, 5, *&c.* to v,
⎨ whoſe Sum is z.
Then for *Lem.* 2. ⎨ 1, 4, 9, 16, 25, *&c.* to v^2,
⎩ whoſe Sum is *A*.

BUT $v^2 = \infty$ (by *Supp.*) Therefore $v = \sqrt{\infty} = \overline{\infty}|^{\frac{1}{2}}$ ſo that $\infty^{\frac{1}{2}}$ is the Value of v in *Lem.* 2*d.* In like manner $\sqrt[3]{\infty}$ or $\infty^{\frac{1}{3}}$ is the Value of v in *Lem.* 3. And univerſally if *e* denote the Exponent of the Power to which each of the Terms of any arithmetical Progreſſion is raiſed, then $v = \sqrt[2]{\infty} = \overline{\infty}|^{\frac{1}{e}}$. in the *Lemma* that gives the Sum of theſe Terms (whoſe laſt is ∞) raiſed to this Power.

Prop. 1. To find the Sum of a Series of Numbers (continued till the laſt Term is ∞) whoſe Terms are the Squares of any arithmetical Progreſſion.

BY *Lem.* 2. the Sum ſought is $A = \frac{v^3 - x^2v}{3x}$

$\frac{v^{2}-xv}{2}$, but in this Caſe $v=\sqrt{\infty}$ or $\infty^{\frac{1}{2}}$ therefore all the Terms, except the higheſt, muſt evaniſh, and conſequently the Sum of the Squares $A=\frac{v^{3}}{3x}=\frac{\overline{\infty^{\frac{1}{2}}}|^{3}}{3x}=\frac{\infty^{\frac{3}{2}}}{3x}=\infty\times\frac{\infty^{\frac{1}{2}}}{3x}$.

Examp. Let x be $=1$, ſo 1, 2, 3, 4, *&c.* to $\sqrt{\infty}$ is the arithmetical Progreſſion, the Sum (A) of whoſe Squares is ſought; I ſay $1+4+9+16$, *&c.* $\infty\times\frac{1}{3}\sqrt{\infty}$, that is, the Sum of the Progreſſion is an infinite Number of equal Parts, each of which is $\frac{1}{3}\sqrt{\infty}$.

Prop. 2. To find the Sum of the Cubes of any arithmetical Progreſſion.

IN this Caſe $v=\sqrt[3]{\infty}$ or $\infty^{\frac{1}{3}}$. Therefore in *Lem.* 3. all the Terms except the higheſt muſt evaniſh, ſo that $B=\frac{v^{4}}{4x}=\frac{1\;\overline{\quad}|}{4x}\infty^{\frac{1}{3}}|^{4}=\frac{\infty^{\frac{4}{3}}}{4x}$ $=\infty\times\frac{\infty^{\frac{1}{3}}}{4x}$.

Ex. Let $x=1$, ſo 1, 2, 3, 4, 5, *&c*, to $\sqrt[3]{\infty}$ is the arithmetical Progreſſion, the Sum (B) of whoſe Cubes is ſought, I ſay $1+8+27+64+125$ *&c.* $=\infty\times\frac{\infty^{\frac{1}{3}}}{3}$

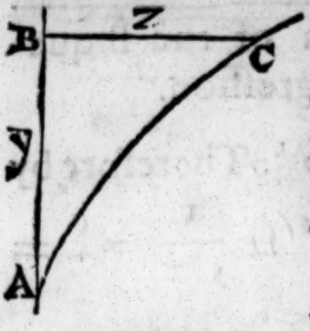
Prop. 3. To find the Sum of the Biquadrates of any arithmetical Progreſſion.

In this Caſe $v = \sqrt[4]{\infty}$ or $\infty^{\frac{1}{4}}$. Therefore by *Lemma* 3*d*. $C = \frac{v^5}{5x} = \frac{1}{5x}\overline{\infty^{\frac{1}{4}}}\big|^{5} = \frac{1}{5x}\infty^{\frac{5}{4}} = \infty \times \frac{\infty^{\frac{1}{4}}}{5x}$

Ex. Let $x = 1$, ſo 1, 2, 3, 4, *&c.* to $\sqrt[4]{\infty}$ is the arithmetical Progreſſion; I ſay then that $1 + 16 + 82 + 256$, *&c.* $= \infty \times \frac{\infty^{\frac{1}{}}}{5}$.

Prop. 4. *Univerſal.* Let e denote the Exponent of any Power, to which the Terms of any arithmetical Progreſſion $a, a+x. a+2x. a+3x$, *&c.* are raiſed, and it is required to find the Sum of the Terms ſo raiſed. I ſay $a^e + \overline{a+x}|^e + \overline{a+2x}|^e + \overline{a+3x}|^e + \overline{a+4x}|^e$ *&c.* $= \infty \times \frac{\infty^{\frac{1}{e}}}{\overline{e+1} \times x}$.

Scholium. From hence may be eaſily deduced the Quadratures of all the Parabola's $z - y^e$ (where z is the Ordinate, y the Abſciſſa, and e an affirmative Number) which is one remarkable Uſe of this Arithmetick of Infi-



Infinites. For if in this Figure $AB = y$, $BC = 2$, and $z = y$, it is plain that if you put $y = 1, 2, 3, 4, 5$, *&c.* ſucceſſively, then will $z = 1^e$, 2^e, 3^e, 4^e, *&c.* ſucceſſively; which ſhews that the Ordinates are the Terms of an arithmetical Progreſſion raiſed to a Power, whoſe Exponent is e, and that (x) the common Difference is 1: But if the firſt Abſciſſa $y = 1$ be infinitely (or indefinitely) ſmall, then the Ordinates will be infinitely near to one another, and the laſt Ordinate BC will be an ∞ Number of theſe Ones. Therefore ſince to find the Area ABC, is, in effect, to find the Sum of the Ordinates, and ſince the Ordinates are 1^e, 2^e, 3^e, 4^e; 5^e *&c.* till you come to the laſt, which is $BC = \infty$. Therefore by Propoſition 4. $ABC = \infty \times \frac{\infty^{\frac{1}{e}}}{e+1}$, that is $ABC = BC \times \frac{\overline{BC}^{\frac{1}{e}}}{e+1}$, the ſame which is found by the ordinary Methods of Quadratures. And note, that the whole Abſciſſa BA denotes the Number of the Terms-

N. B. *The Quadrature of all Sorts of Curves expreſſed by one Term thus aſſigned, it is eaſy by the Method of Aſſumptions of Series's to extend this Method to all Sorts of Quadratures hitherto diſcovered.*

AD-

ADDITIONS.

I have ſhewn in *p.* 160. *&c.* that mean Proportionals between 1 and ∞ are neither finite nor infinite. For a farther Illuſtration of what I have ſaid upon this Head, let us conſider what theſe Progreſſions are, whoſe Sum is more than finite, and leſs than infinite.

Prop. 1. $\sqrt{\infty}$ is a Mean between 1 and infinite; but $\infty = \frac{1}{1-1}$, therefore $\sqrt{\infty} = \frac{1}{\sqrt{1-1}} = \overline{1-1}\big|^{\frac{1}{2}}$ which by Sir *Iſaac Newton's* Theorem gvies $\sqrt{\infty} = \overline{1-1}\big|^{-\frac{1}{2}} = 1 + \frac{1}{2} + \frac{1\times3}{2\times4} + \frac{1\times3\times5}{2\times4\times6} + \frac{1\times3\times5\times7}{2\times4\times6\times8}$, *&c.* that is $\sqrt{\overline{+1+1}}$, *&c.* $= 1 + \frac{1}{2} + \frac{1\times3}{2\times4} + \frac{1\times3\times6}{2\times4\times6} + \frac{1\times3\times5\times7,}{2\times4\times6\times8}$ *&c.*

AND ſo (by ſquaring each Side of this Equation) you wiil find it in Fact to be $1 + 1 + 1$, *&c.* $= 1 + 1 + 1$, *&c.* From hence it appears that the Progreſſion, whoſe Sum (being neither finite nor infinite) is $\sqrt{\overline{\infty}}$ conſiſts of Fractions, whoſe Numerators are the Products of the continual Multiplication of the arithmetical Progreſſion 1, 1, 3, 5, 7, 9, *&c.* and the Denominators are the Products of the Multiplications of the Terms of this arithmetical Progreſſion 2, 4, 6, 8, 10, *&c.*

Corol. In like Manner you may find $\sqrt[3]{\infty}$ $\sqrt[5]{\infty}$ *&c.* by reſolving $\frac{1}{\sqrt[3]{1-1}} = \overline{1-1}\big|^{\frac{1}{3}}$ $\frac{1}{\sqrt[5]{1-1}} = \overline{1-1}\big|^{\frac{1}{5}}$ *&c.* into a Series by Sir *Iſaac Newton*'s Theorems. So

$$\sqrt[3]{\infty} = \frac{1}{\sqrt[3]{1-1}} = \overline{1-1}\big|^{\frac{1}{3}} = 1 + \tfrac{1}{3} + \frac{1 \times 4}{3 \times 6} + \frac{1 \times 4 \times 7}{3 \times 6 \times 9}$$

&c. where the Numerators and Denominators are in arithmetical Progreſſions, whoſe common Difference is 3.

$$\sqrt[5]{\infty} = \frac{1}{\sqrt[5]{1-1}} = \overline{1-1}\big|^{\frac{1}{5}} \quad 1 + \tfrac{1}{3} + \frac{1 \times 6}{5 \times 10} + \frac{1 \times 6 \times 11}{5 \times 10 \times 15}$$

&c. $\sqrt[e]{\infty} = \frac{1}{\sqrt[e]{1-1}} = \overline{1-1}\big|^{\frac{1}{e}}$

$$= 1 + \frac{1}{e} + \frac{1 \times \overline{e+1}}{e \times 2e} + \frac{1 \times \overline{e+1} \times \overline{2e+1}}{e \times 2e \times 3e} + \frac{1 \times \overline{e+1} \times \overline{2e+1} \times \overline{3e+1}}{e \times 2e \times 3e \times 4e} + \frac{1 \times \overline{e+1} \times \overline{2e+1} \times \overline{3e+1} \times \overline{4e+1}}{e \times 2e \times 3e \times 4e \times 5e}$$

&c.

Prop. 2. Let 1 be the firſt, r the ſecond, and v the laſt Term of a geometrical Progreſſion 1, r^1, r^2, r^3, r^4, *&c.* and t the Number of Terms, then $v = r^{t-1} = \frac{r^t}{r}$, which gives $r^t = rv$.

Corol. Let τ be the Number of Terms, and v the laſt Term in the like Progreſſion 1, s, s^2, s^3, s^4, *&c.* then $v = \overline{s}|^{\tau-1}$.

Corol. 2. From theſe two Values of v, I have this Equation $r^{t-1} = s^{\tau-1}$; whence it appears that if s be greater than r, then will $\tau < t$, that is, in any two Progreſſions whoſe firſt Terms are the ſame, that, whoſe ſecond Term s is greater than r the ſecond Term of the other, will ſooner arrive at any given Number v.

Examp. Let $r = 2$, then 1, 2, 4, 8, 16. Let $s = 4$, then 1, 4, 16, that is, there are 5 Terms (or $t = 5$) in arriving at 16 in the firſt Progreſſion, whereas in the ſecond ($\tau = 3$) it arrives at 16 in the 3*d* Term.

Problem. Having r, t, s, to find τ, in which they ſhall both have the ſame laſt Term v.

Sol. $s^{\tau-1} = r^{t-1}$ (by *Corol.* 2.) this turned into a logarithmical Equation is $\overline{\tau - 1} \times ls = \overline{t-1} \times lr$, which reduced gives $\tau = \frac{tlr + ls - lr}{ls} = \frac{tlr - lr}{ls} + 1$. *Q. E. I.*

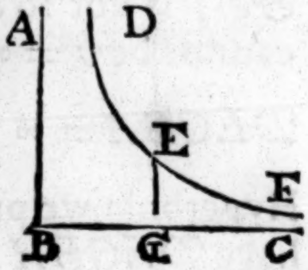
Corol. Becauſe by *Prop.* 2. $r_t = r\,v$, therefore $t\;l\,r = l\,r\,v$, ſo ſubſtituting $l\,r\,v$ in place of $t\,l\,r$, we ſhall have $\tau = \frac{l\,r\,v - l\,r + l\,s}{l\,s} = \frac{l\,r\,v}{l\,s} - \frac{l\,r}{l\,s} + 1$.

Corol. 2. Becauſe by *Corol.* 1. $t\,l\,r = l\,r\,v$, therefore $t = \frac{l\,r\,v}{l\,r}$. And conſequently $t - \tau = \frac{l\,r\,v}{l\,r} - \frac{l\,r\,v}{l\,s} + \frac{l\,r}{l\,s} - 1$.

But by the known Property of Numbers we have $l\,r + l\,v = l\,r\,v$, therefore $t - \tau = \frac{lr + l\,v}{lr} - \frac{lr - lv}{l\,s} + \frac{l\,r}{l\,s} - 1$ that is $t - \tau = 1 + \frac{l\,v}{l\,r} - \frac{l\,v}{l\,s} - 1$, that is, $t - \tau = \frac{l\,v}{l\,r} - \frac{l\,v}{l\,s}$.

Corol. 3. Let the laſt Term v be infinite or ∞, then $t - \tau = \frac{l : \infty}{l\,r} - \frac{l : \infty}{l\,s}$; which ſhews how much ſooner the Progreſſion 1, s, s^2,

s^3, *&c.* will arrive at ∞, than 1, r, r^2, r^3, *&c.* that is, how many more Terms there is in 1, r, r^2, r^3, *&c.* than in 1, s, s^2, s^3, *&c.* when the laſt in both is ∞.



Prop. 3. Let $BG = x$ $GE = y$; $y\,x = 1$, where s is any Number greater than Unity. Then by the *Calculus Integralis*, we have $DEGBA = \dfrac{\overline{x}\,|^{1-s}}{1-s}$, ſo putting $x = 1$. we have

$$DEGBA = \frac{1}{1-s}.$$

But $\dfrac{1}{1-s} = 1 + s + s^2 + s^3 + s^4$, *&c.* $= DEGBA$.

But calling v the laſt Term of this aſcending geometrical Progreſſion $1 + s + s^2 + s^3 +$ *&c.* we ſhall (as is ſhewn in Page 135. of this Chapter) have $1 + s + s^2 + s^3 + s^4 +$ *&c.* $= \dfrac{sv}{s-1}$, therefore $DEGBA = \dfrac{sv}{s-1}$. So that we have three different Expreſſions of the Area $DEGBA$, *viz.* $\dfrac{1}{1-s}$,

$\frac{s\,v}{s-1}$, and the Series $1+s+s^{2}+s^{3}+s^{4}$, *&c.*

But what this Number v is (which denotes the laſt Term of the Progreſſion $1+s+s^{2}+s^{3}$, *&c.*) is not ſo eaſy to determine; certainly it cannot be $v=\infty\times 1$ or $1+1+1$, *&c.* For then $DEGBA=\infty\times\frac{s}{s-1}$; therefore if in one Hyperbola we put $s=2$, (*ſcil.* $y\,x^{2}=1$) we have $DEGBA=\infty\times 2$; and putting in another $1=3$ (*ſcil.* $y\,x^{3}=1$) we have $DEGBA=\infty\times\frac{3}{2}$; now this would make the Area of the Hyperbola $y\,x^{2}=1$ *ſcil.* $\infty\times 2$ greater than the Area of the Hyperbola $y\,x^{3}=1$ *ſcil.* $\infty\times\frac{3}{2}$; (becauſe $2<\frac{3}{4}$) but it is eaſy to demonſtrate, that in the Hyperbola's $y\,x^{s}=1$. the greater we ſuppoſe the Exponent s, ſo ſhall the Area $DEGBA$ (adjacent to BG) be the greater, and conſequently the laſt Term v of the Series $1+s+s^{2}+s^{3}$, *&c.* cannot be ∞ or $1+1+1+1+1$, *&c.* for upon that Value of v, the Expreſſion $\frac{s\,v}{s-1}$ would make the Area $DEGBA$ to decreaſe as s did increaſe.

To diſcover the Value of v, we have $DEGBA=\frac{s\,v}{s-1}=\frac{1}{1-s}$; which gives v $=\frac{s-1}{s-s}{}^{2}=\frac{s-1}{s}\times\frac{1}{1-s}=\frac{1}{-s}$.

HAVING found $v = \frac{1}{-s}$, we ſee that in two different Hyperbola's (*Ex. gr.* $yx^2 = 1$, and $yx^3 = 1$,) v cannot have the ſame Value; for in the former $v = \frac{1}{-1}$ which is leſs then $v = \frac{1}{-3}$, in the other.

Secondly. From the Value of $v = \frac{s-1}{s} \times \frac{1}{1-s}$ we ſee that the laſt Term v is equal to the Sum of the Series (*viz.* $1 + s + s^2 + s^3$ *&c.*) multiplied upon $\frac{s-1}{s}$ for $\frac{1}{1-s}$ is $= 1 + s + s^2 + s^3$ *&c.*

Corol. I ſay the laſt Term v cannot be $\infty \times n$, take what Number you will for n (except in the Caſe of the common Hyperbola, where $s = 1$.) For if it were poſſible, let $v = \infty \times n$; then muſt $\frac{s-1}{s-s^2} = \infty \times n$, therefore $s - 1 = \infty \times ns^2 - ns^2$, which gives $1 = \infty \times \frac{ns - ns^2}{s-1}$, and conſequently $\frac{1}{\infty} = \frac{ns = ns^2}{s-1}$, but $\frac{1}{\infty} = 0$, therefore $\frac{ns - ns^2}{s-1} = 0$, which give $s = 1$. *Q. E. D.*

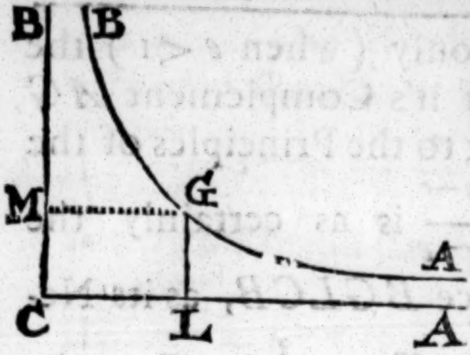
Corol. 2. It is evident that v is ſome Power of s, let the Exponent of that Power be n, *ſcil.* $v = s^n$, but $\frac{sv}{s-1} = \frac{1}{1-s}$, therefore

$\frac{s \times s^n}{s-1} \pm \frac{1}{1-s}$, *id eſt*, $\frac{s^n+1}{s-1} = \frac{1}{1-s}$, therefore $s^{n-1} = \overline{s-1} \times \frac{1}{1-s}$, which reduced to a logarithmic Equation gives $\overline{n+1} \times l : s = l : \overline{s-1} + l : \frac{1}{1-s}$, which gives $n = \frac{l : \overline{s-1} + l : \frac{1}{1-1}}{l : s} - 1$ *Q. E. I,*

An ANSWER *to Mr.* Varignon's *Reflections upon Spaces greater than Infinite.*

LET BC, AC making a right Angle at C be the Aſymptotes of any Hyperbola BGA; the Abſciſſe $CL = x$, and Ordinate $LG = y$; and the general Equation to all Hyperbola's $yx^e = 1$. Then becauſe $y = \frac{1}{x^e} = x^{-e}$, we have $y\,dx = x^{-e}\,dx$, therefore $\int : y\,dx = \int : x^{-e}\,dx$; but by the *Calculus Integralis* $\int : x^{-e}\,dx = \frac{x^{1-e}}{1-e}$, therefore $\int : y\,dx = \frac{x^{1-e}}{1-e}$; But $\int : y\,dx = BGLCB$ therefore $\frac{x^{1-e}}{1-e} = BGLCB$;

THIS



THIS Concluſion (though deduced naturally from Principles which Mr. *Varignon* acknowledges to be true) he denies, aſſuring us that it is only the negative Expreſſion of the Area $AGLA$ or Complement of $BGLCB$, when $e < 1$. But (without any Regard to the Ratio of e to 1) let us conſider the algebraical Expreſſions of theſe two Spaces $BGLCB$, and $AGLA$; we have already found $BGLCB = \frac{x^{-e}}{1-e}$; and calling the Abſciſſe $CM = y$, and Ordinate $MG = x$, we ſhall from the Equation $y x^{e} = 1$, find $ACMGA = \frac{\overline{e \times y}|^{\frac{e-1}{e}}}{e-1}$ but $y^{\frac{e-1}{e}} = x^{1-e}$; therefore $ACMGA = \frac{e \times x^{i-e}}{e-1}$, from which if you ſubſtract $CMGL = yx = x^{1-e}$, the Remainder will be (as Mr. *Varignon* ſays) $AGLA = \frac{x^{1-e}}{e-1}$, which is indeed the Negative of $\frac{x^{1-e}}{1-e}$.

But his Concluſion is falſe and groundleſs, *viz.* That $\frac{x^{1-e}}{1-e}$ is not the Expreſſion of the

Space *BGLCB*, but only (when $e < 1$) the negative Expreſſion of it's Complement *AGLA*. For (according to the Principles of the *Calculus Integralis*) $\frac{x^{1-e}}{1-e}$ is as certainly the Expreſſion of the Space *BGLCB*, as its Negative $\frac{x^{1-e}}{e-1}$ is the Expreſſion of its Complement *AGLA*. And this is ſuitable to Nature's Proceedings, *viz.* to do things in the ſimpleſt and moſt general Method, and conſequently to give one Expreſſion of the Area of a Figure, and of its Complement, when it can be done. And may it not (with as good Reaſon) be ſaid that $\frac{x^{1-e}}{e-1}$ is not the Expreſſion of the Space *AGLA*, but of its Complement *BGLCB*; and not only it may be ſaid, but Mr. *Varignon* muſt ſay ſo in all the innumerable Caſes where $e > 1$; and to ſay it is not when $e < 1$, and it is when $e > 1$; what is this but to ſay backward and forward, according as it will anſwer his Conceit of denying that there are Spaces greater than Infinite? But to put this Controverſy beyond all Diſpute, let us reſolve theſe two finite Expreſſions of the Spaces *BGLCB*, and *AGLA* into infinite Series's,

I. $\frac{x^{1-e}}{1-e} = x^{1-e} \times 1 + e + e^2 + e^3 + e^4$, &c. when you begin the Diviſion with 1, as here it naturally ſhould.

II. $\frac{x^{1-e}}{1-e} = x^{1-e} \times -\frac{1}{e} - \frac{1}{e^2} - \frac{1}{e^3} - \frac{1}{e^4}$, &c. when you begin the Diviſion with $-e$.

III. $\frac{x^{1-e}}{e-1} = x^{1-e} \times \frac{1}{e} + \frac{1}{e^2} + \frac{1}{e^3} + \frac{1}{e^4}$, &c. when you begin the Diviſion with e, as here it naturally ſhould.

IV. $\frac{x^{1-e}}{e-1} = x^{1-e} \times -1 - e - e^2 - e^3 - e^4$, &c. when you begin the Diviſion with -1.

Thus it appears, that each of theſe two Expreſſions give two Series, of which the firſt is the Area directly ſought, and the other the negative Expreſſion of its Complement; ſo

$$\left\{\begin{aligned} &BGLCB = \frac{x^{1-e}}{1-e} = x^{1-e} \times 1 + e + e^2 + e^3 + e^4 + \text{\&c.} \\ &-AGLA = \frac{x^{1-e}}{1-e} = x^{1-e} \times -\frac{1}{e} - \frac{1}{e^2} - \frac{1}{e^3} - \frac{1}{e^4} - \text{\&c.} \end{aligned}\right.$$

$$AGLA = \frac{x^{1-e}}{e-1} = x^{1-e} \times \frac{1}{e} + \frac{1}{e^2} + \frac{1}{e^3} +$$

$\frac{1}{e^4} + \&c.$

$$-BGLCB = \frac{x^{1-e}}{e-1} = x^{1-e} \times -1 - e - e^2 - e^3 - e^4 - \&c.$$

Theſe are plain and obvious Concluſions, deduced from the known and common Operations of Arithmetick, and utterly overthrow all that Mr. *Varignon* alledges againſt Spaces greater than Infinite.

For without any Limitation (except $e \vartriangleleft 1$) he ſays that $\frac{x^{1-e}}{1-e}$ is not the Expreſſion of *BGLCB*, but the Negative of the Space *AGLA*. And if ſo, then (by Series I.) it will follow that $x^{1-e} \times 1 + e + e^2 + e^3 + e^4$, &c. is the negative Expreſſion of the Space *AGLA*, and this I ſay implies two manifeſt Abſurdities; (1ſt) That a Sum of affirmative Quantities is negative; and (2dly,) That the Sum of an infinite geometrical Progreſſion, whoſe Terms are continually increaſing, is equal to a finite Quantity, *i, e.* That Affirmative is Negative, and Infinite is Finite.

That which ſeems to ſhock Mr. *Varignon* is, that there ſhould be Spaces greater than Infinite; for he tells us expreſſly, that he looks upon this as a Contradiction. But this Difficulty will ſoon evaniſh, if he conſiders what is the ſimple Idea of Infinite; and becauſe he confeſſes that in the *Apollonian* Hyperbola *BGA*, the Space *BGLCB* is Infinite; therefore let us take the arithmetick Value of *BGLCB* for infinite, but in that Hyperbola $e = 1$, and therefore (if we put likewiſe $x = 1$) we ſhall by Series I. have $BGLCB = 1 + 1 + 1 + 1 + 1$ *&c.* that is $BGLCB = \infty \times 1$ or an infinite Number of Units. So that when we ſay $2 + 2 + 2 + 2$, *&c.* or $\infty \times 2$ is more than infinite, no more is meant, but that it is greater than ſimple Infinite or $\infty \times 1$; *i. e.* (taking $\infty \times 1$ for the Unit, by which we meaſure Quantities that are infinite) $\infty \times 2 \lhd \infty \times 1$, or $2 \lhd 1$.

This will be yet plainer in the Caſe of Finites. It is certain no Quantity can be leſs than Finite (as none can be greater than Infinite) which we ſhall denote by $\odot \times 1$ or 1; when I ſay then, that $\odot \times \frac{1}{2}$ is leſs than $\odot \times 1$, I mean no more but that $\frac{1}{2} \rhd 1$; and becauſe I have defined the Idea of finite by $\odot \times 1$ or 1, I may very properly ſay, that $\odot \times \frac{1}{3}$ or $\frac{1}{2}$ is leſs than a finite Quantity.

September 23d, 1713.

FINIS.

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